①

$y=ax^2+bx+c \longrightarrow (3,1)$ نقطه $\rightarrow 1=9a+3b+c \rightarrow 1=9a+4a+9+a \rightarrow -8=14a \rightarrow a=-\frac{1}{2}$

$\frac{-b}{2a}=-1 \rightarrow -b=-2a \rightarrow b=2a \rightarrow b=-1$

$\frac{-\Delta}{4a}=\frac{-b^2+4ac}{4a}=9 \rightarrow \frac{-4a^2+4ac}{4a}=\frac{4a(-a+c)}{4a}=9 \rightarrow c=9+a \rightarrow c=\frac{19}{2}$

پس $\rightarrow \boxed{y=-\frac{1}{2}x^2-x+\frac{19}{2}}$

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④

$\Delta>0 \rightarrow b^2-4ac=m^2-4(2)(m+4)>0 \rightarrow m^2-8m-32>0 \rightarrow (m-12)(m+4)>0$

$2m+12$　　12　　-4

$\frac{-b}{2a}>0$ ، $\frac{c}{a}>0$

-4　　12

$+ \quad \phi \quad - \quad \phi \quad +$

ج.پ $\rightarrow (-\infty,-4]\cup[12,+\infty)$

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③

$-\frac{b}{a}=\frac{a}{c} \rightarrow -bc=a^2$　　$(1-2m)(2-m)=9 \rightarrow 2-m-4m+2m^2=9 \rightarrow 2m^2-5m-7=0$

$m^2-5m-14=0 \rightarrow (m-7)(m+2)=0 \rightarrow m=\frac{7}{2}, m=\frac{-2}{2}=-1$

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④

$x=x^2-4 \rightarrow x^2-x-4=0 \rightarrow \frac{-b}{a}=\frac{1}{1}=1$ مج　$\frac{c}{a}=\frac{-4}{1}=-4$ ضرب

$\frac{1}{x_1}+\frac{1}{x_2}+2x_1^3+2x_2^3 \rightarrow x_1^3+x_2^3+\frac{x_1^3+x_2^3}{x_1^3x_2^3}=5-\frac{1}{4}=\frac{20-1}{4}=\frac{19}{4}$ ـ مج

$x_1^3+x_2^3=(x_1+x_2)(x_1^2-x_1x_2+x_2^2)=(1)(x_1^2+x_2^2+4)=5$

$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=1+8=9$

$x=x^2-4 \rightarrow x_1=x_1^2-4$ ، $x_2=x_2^2-4$

$x_1^2=x_1+4$　　$x_2^2=x_2+4$

ضرب

$\left(\frac{1}{x_1}+\frac{1}{x_2}\right)\left(x_1^3+\frac{1}{x_2^3}\right)=(x_1x_2)^3+x_1^2+x_2^2+\frac{1}{x_1^3x_2^3}=-64+9-\frac{1}{4}=-55-\frac{1}{4}=\frac{-220-1}{4}=\frac{-221}{4}$

$\boxed{x^2-\frac{19}{4}x-\frac{221}{4}}$

(۵)

ضرب → $x\sqrt[3]{x} - \sqrt[3]{x^2} + 1 - \frac{1}{\sqrt[3]{x^2}} + \sqrt[3]{x^2} - 1 = x\sqrt[3]{x} - \frac{1}{\sqrt[3]{x^2}} = 2\sqrt[3]{x}$ $(\sqrt[3]{x^2})$

$\to x^2 - 1 = 2x \to x^2 - 2x - 1 = 0$ $\quad \Delta = 4 + 4 = 8$ $\quad \frac{2 \pm \sqrt{8}}{2} \to \frac{2+\sqrt{8}}{2} + \frac{2-\sqrt{8}}{2} = \frac{4}{2}$ (۲)

مجموع ریشه‌ها $= -\frac{b}{a} = \frac{2}{1} =$ (۲)

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(۶)

$3x^2 - ax + 4 = 0 \to \frac{c}{a} = \frac{4}{3}$

$x_1, x_2 \to x_2 = 3x_1 \to (x_1)(3x_1) = 3x_1^2 = \frac{4}{3} \to x_1^2 = \frac{4}{9} \to x_1 = \pm\frac{2}{3}$

$3(\frac{4}{9}) - \frac{2}{3}a + 4 = 0 \to \frac{4}{3} - \frac{2}{3}a + 4 = 0 \to \frac{16}{3} = \frac{2}{3}a \to a = 8$

$3(\frac{4}{9}) + \frac{2}{3} + 4 = 0 \to \frac{4}{3} + 4 = -\frac{2}{3}a \to \frac{16}{3} = -\frac{2}{3}a \to a = -8$

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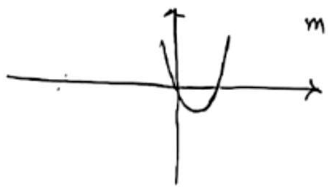
(۷)

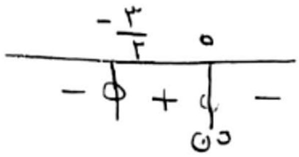
$y = ax^2 + (3 + 2a)x$

$\frac{-b}{a} \geq 0 \Rightarrow \frac{-3-2a}{a} \geq 0$ $\quad [-\frac{3}{2}, 0) \to$ در منفی‌ها است / از ناحیه ۳ تا ۰

min $\quad a > 0$



$-3 - 2a = 0$
$-3 = 2a$
$a = -\frac{3}{2}$
$a = 0$



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(۸)

$\frac{-b}{2a} \to \frac{-a}{2} = \frac{2}{-2} \to 4 = 2a \to a = 2 \to y = -x^2 + 2x + b$

$1 = x^2 + 2x - 2 \to x^2 + 2x - 3 = 0 \to (x+3)(x-1) = 0 \to x = 1,\ x = -3$

$y = -x^2 - 2x + b$

$1 = -1 + 3 + b \to b = 4$ $\quad$ پس $\to ab = 2 \times 4 = \boxed{8}$

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(۹)

جمع ریشه‌ها یک واحد بیشتر $\to \boxed{\frac{-b}{a}} = \frac{a}{2} = \frac{-a}{2a} + 1 \to \frac{a}{2} = \frac{-a+2a}{2a} \to \frac{a}{2} = \frac{a}{2a} \to 2a^2 = 2a \to a = 1$

$2x^2 + x - 6 = 0 \to x^2 + x - 12 = 0 \to (x+4)(x-3) = 0 \to x = -\frac{4}{2}, \frac{3}{2} \to -2, 1/5$

پس ریشه‌های اولیه $\leftarrow -1/5, 2 \leftarrow (x^2 - 0/5x - 3)2 = 2x^2 - x - 6$ $\quad -6 = b$

$[\frac{-6 \times 1}{4}] = [\frac{-6}{4}] = \boxed{-2}$

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(۱۰)

$x^2 + 2x - 3m = x^2 + 6x + m$

$0 = 4x + 4m \to 4x = -4m \to x = -m$

$x^2 + 6x + m = 0 \to \frac{c}{a} = m \to -m \times x = m \to x = -1$

$x^2 + 2x - 3m = 0 \to \frac{c}{a} = -3m \to -m \times x = -3m \to x = 3$

$\Big\}\ 3 - (-1) = \boxed{4}$