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$-\frac{b}{2a}=-1 \to b=2a \to ax^2+2ax+c=y$

1) $(-1,9) \to a-2a+c=9 \quad , (3,1) \to 9a+6a+c=1$

$\to a-2a+c-9a-6a-c=9-1 \to -16a=8 \to a=-\frac{1}{2} \to b=-1$

$\to -a+c=9 \to c=\frac{17}{2}=8/5 \to \boxed{y=-\frac{1}{2}x^2-x+8/5}$

2) $\Delta>0 \to m^2-8(m+6)>0 \to (m-12)(m+4)>0 \to \frac{-4 \quad 12}{+ \; - \; +} \to (-\infty,-4)\cup(12,+\infty)$

$S>0 \to \frac{-m}{2}>0 \to m<0 \quad , \quad P>0 \to \frac{m+6}{2}>0 \to m+6>0 \to m>-6$

$\xrightarrow{\cap} \boxed{m\in(-6,-4)}$

3) $-\frac{b}{a}=\frac{1}{\frac{c}{a}} \to -\frac{b}{a}=\frac{a}{c} \to a^2=-bc \to 9=(-2m+1)(2-m)$

$\to 9=-4m+2m^2+2-m=2m^2-5m+2 \to 2m^2-5m-7=0 \to$

$m^2-5m-14=0 \to (m-7)(m+2)=0 \to m=\frac{7}{2} \quad , m=-\frac{2}{2}=-1$

$\Delta>0 \begin{cases} m=-1 \to 3x^2-3x+1=0 \to \Delta=9-(1\times3\times4)=9-12=-3 \quad \times \\ \boxed{m=3/5} \to 3x^2-6x-1/5=0 \to \Delta=36+(3\times1/5\times4)=36+18=54 \quad \checkmark \end{cases}$

4) $x_1+x_2=1 \quad , x_1x_2=-4$

$\to S'=x_1^2+\frac{1}{x_2}+x_2^2+\frac{1}{x_1}=(S^2-2SP)+\frac{x_1+x_2}{x_1x_2}=(1^2+12)-\frac{1}{4}=\frac{51}{4}$

$\to P'=(x_1^2+\frac{1}{x_1})(x_2^2+\frac{1}{x_1})=(x_1x_2)^2+\underbrace{x_1^2+x_2^2}_{(x_1+x_2)^2-2x_1x_2}+\frac{1}{x_1x_2}=$

$=(-4)^3+(1)^2-2(-4)+\frac{1}{-4}=-64+1+8-\frac{1}{4}=\frac{-221}{4}$

$\boxed{(x')^2-\frac{51}{4}-\frac{221}{4}=0}$

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$(\sqrt[3]{x^2}+\frac{1}{\sqrt[3]{x^2}}+1)(\sqrt[3]{x^2}-1)=3\sqrt[3]{x} \xrightarrow{\times\sqrt[3]{x^2}} (\sqrt[3]{x^4}+1+\sqrt[3]{x^2})(\sqrt[3]{x^2}-1)$

$= x^2 \cancel{-\sqrt[3]{x^4}} \cancel{+\sqrt[3]{x^2}} - 1 \cancel{+\sqrt[3]{x^4}} \cancel{-\sqrt[3]{x^2}} = x^2-1 = 3x \to x^2-3x-1=0$

$\to S = x_1 + x_2 = 3$

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$3x^2 - ax + 4 = 0 \qquad x_1 = 3x_2$

$S = 3x_2 + x_2 = 4x_2 = \frac{a}{3} \to 12x_2 = a \to 12 \times \pm\frac{2}{3} = \pm 8$

$P = 3x_2 \times x_2 = 3x_2^2 = \frac{4}{3} \to \sqrt{9x_2^2} = \sqrt{4} \to 3x_2 = \pm 2 \to x_2 = \pm\frac{2}{3}$

$8 - (-8) = \boxed{16}$

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$\Delta > 0$ و [illegible] $\to S > 0$

$\frac{-b}{a} = \frac{-3-2a}{a} > 0 \to -3-2a=0 \to -3=2a \to a=-\frac{3}{2}$

$a = 0$

$-\frac{3}{2} \quad 0$

$- \quad + \quad -$

$\to -\frac{3}{2} < a < 0 \to (-\frac{3}{2}, 0) \cap (0, +\infty) = \emptyset \to$ [illegible]

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$y = x^2 + ax - 2 \xrightarrow{\text{رأس}} x = -\frac{a}{2}$

$y = -x^2 - 2x + b \xrightarrow{\text{رأس}} x = -1$

$-\frac{a}{2} = -1 \to -a = -2 \to a = 2$

$(\ ,1), (s,1) \to s^2 + 2s - 2 = 1 \to s^2 + 2s - 3 = 0 \to (s+3)(s-1)=0 \to s=-3, 1$

$\to s = -3 \to (-3,1) \to -9+6+b=1 \to b=4$

$s = 1 \to (1,1) \to -1-2+b=1 \to b=4$

$a \times b = 2 \times 4 = \boxed{8}$

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(9)

① $2x^2 - ax + b = 0 \longrightarrow S = \frac{a}{2}$, $P = \frac{b}{2}$

② $2ax^2 + ax - 6 = 0 \longrightarrow S = \frac{-a}{2a} = -\frac{1}{2}$,

$S_1 = S_2 + 1 \to \frac{a}{2} = -\frac{1}{2} + 1 = \frac{1}{2} \to a = 1$

① $\to 2x^2 - x + b = 0$, ② $\to 2x^2 + x - 6 = 0 \longrightarrow x^2 + x - 12 = 0 \to (x+4)(x-3) = 0$

$\longrightarrow x_1 = \frac{-4}{2} = -2$, $x_2 = \frac{3}{2} \longrightarrow x_1 x_2 = -2 \times \frac{3}{2} = -3$

$\longrightarrow -3 = \frac{b}{2} \longrightarrow b = -6$

$\to \left[\frac{ab}{4}\right] = \left[\frac{-6\times 1}{4}\right] = \left[\frac{-6}{4}\right] = \left[\frac{-3}{2}\right] = \boxed{-2}$

(10)

$\alpha =$ ریشه مشترک $\to \alpha^2 + 6\alpha + m = 0$
$\alpha^2 + 2\alpha - 3m = 0$ $\longrightarrow \alpha^2 - \alpha^2 + 6\alpha - 2\alpha + m + 3m = 0$

$\to 4\alpha = -4m \to \alpha = -m \to -\alpha = m \to \alpha^2 + 6\alpha + m = 0 \to \alpha^2 + 6\alpha - \alpha = 0$

$\to \alpha^2 + 5\alpha = 0 \to \alpha = 0$ ✗

$\alpha = -5 \to m = 5$ ✓ $\longrightarrow x^2 + 6x + 5 = (x+5)(x+1) = 0$ → $x = -5$, → $x = -1$

$x^2 + 2x - 15 = (x+5)(x-3) = 0$ → $x = -5$, → $x = 3$

$\to -1 , 3$ غیرمشترک اختلاف $= 3 - (-1) = 3 + 1 = \boxed{4}$