تکلیف شماره ۲۳

Subject:

Year: Month: Date:

سارینا زارع

دهم رشته A

۱- $A(-1,9)$ ، $y=ax^2+bx+c$ ، گذرا از $(3,1)$

$\frac{-b}{2a}=-1 \rightarrow \frac{b}{2a}=+1 \rightarrow b=2a$ ★

$A(h,k) \rightarrow$ راس سهمی $\rightarrow k=y(h)=ah^2+bh+c$

$y(-1)=9=a(-1)^2+b(-1)+c \Rightarrow a-b+c=9 \xrightarrow{★} a-2a+c=9$ ۵

$\hookrightarrow c=9+a$

$\hookrightarrow y(3)=1 \rightarrow a(3)^2+b(3)+c=1 \rightarrow 9a+3b+c=1 \rightarrow 9a+3(2a)+(9+a)=1$

$\rightarrow 16a+9=1 \rightarrow 16a=-8 \rightarrow a=\frac{-1}{2} \Longrightarrow b=2a=2\times\frac{-1}{2}=-1$

$c=9+a=9+(-\frac{1}{2})=\frac{17}{2}$

معادله سهمی : $\frac{-1}{2}x-x+\frac{17}{2}$

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۲- $2x^2+mx+m+4=0$

در رشته مثبت : $\Delta>0 \rightarrow m^2-8m-48>0 \rightarrow (m-12)(m+4)>0$

★۱ $m>12$ یا $m<-4$

$p>0 \rightarrow \frac{c}{a}>0 \rightarrow \frac{m+4}{2}>0 \rightarrow m>-4$ ★۲

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$S>0 \rightarrow \frac{-b}{a}>0 \rightarrow \frac{-m}{2}>0 \rightarrow m<0$ ★۳

★۱ ، ★۲ ، ★۳ → اشتراک : $-4<m<-4$

۳- $3x^2+(2m-1)x+2-m=0$

در رشته حقیقی : $\Delta>0$ ، $S=\frac{1}{p} \rightarrow -\frac{2m-1}{3}=\frac{3}{2-m} \rightarrow$ ۲۰

$\rightarrow -(2m-1)(2-m)=9 \rightarrow (2m-1)(m-2)=9 \rightarrow 2m^2-4m-m+2=9$

$\rightarrow 2m^2-5m-7=0 \rightarrow m=\frac{5\pm\sqrt{25+56}}{4}=\frac{5\pm9}{4} \rightarrow m=\frac{14}{4}=\frac{7}{2}$

بررسی شرط دلتا : if $m=\frac{7}{2} \rightarrow \Delta=4^2-4\times3\times(-\frac{3}{2})=36+18$! $m=\frac{-4}{4}=-1$

$\Delta=54>0$ ✓

if $m=-1 \rightarrow \Delta=(-3)^2-4\times3\times3=9-36=-27<0$ ✗

$m=\frac{7}{2}$

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$$x = x^2 - 4 \rightarrow x^2 - x - 4 = 0 \rightarrow S = \frac{-b}{a} = 1, \quad P = \frac{c}{a} = -4$$

$$x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 = 1^2 - 2(-4) = 1 + 8 = 9$$

$$x_1^3 + x_2^3 = (x_1 + x_2)^3 - 3x_1x_2(x_1 + x_2) = 1^3 - 3(-4)(1) = 1 + 12 = 13$$

$$y_2 = x_2^3 + \frac{1}{x_1}, \quad y_1 = x_1^3 + \frac{1}{x_2}$$

$$y_1 + y_2 = x_1^3 + x_2^3 + \frac{1}{x_2} + \frac{1}{x_1} = (x_1^3 + x_2^3) + \frac{x_1 + x_2}{x_1 \times x_2} = 13 - \frac{1}{4} = \frac{51}{4}$$

$$y_1 \cdot y_2 = \left(x_1^3 + \frac{1}{x_2}\right)\left(x_2^3 + \frac{1}{x_1}\right) = x_1^3x_2^3 + x_1^3 \times \frac{1}{x_1} + \frac{1}{x_2} \times x_2^3 + \frac{1}{x_2} \times \frac{1}{x_1}$$

$$= (x_1x_2)^3 + x_1^2 + x_2^2 + \frac{1}{x_1x_2} = (-4)^3 + 9 - \frac{1}{4} = -64 + 9 - \frac{1}{4} = \frac{-221}{4}$$

$$t^2 - (y_1 + y_2)t + y_1y_2 \Rightarrow t^2 - \frac{51}{4}t - \frac{221}{4} = 0 \xrightarrow{\times 4} \boxed{4t^2 - 51t - 221 = 0}$$

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$$\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x} \rightarrow t = \sqrt[3]{x}, \quad t^2 = \sqrt[3]{x^2}$$

$$\underbrace{\left(t^2 + \frac{1}{t^2} + 1\right)(t^2 - 1)}_{t^4 - \frac{1}{t^2}} = 2t \rightarrow t^4 - \frac{1}{t^2} = 2t \xrightarrow{\times t^2} t^6 - 2t^3 - 1 = 0$$

بسط

از بین بردن مخرج

$$u = t^3 \rightarrow u^2 - 2u - 1 = 0 \rightarrow u = \frac{2 \pm \sqrt{8}}{2} = \frac{2(1 \mp \sqrt{2})}{2} = \boxed{1 \pm \sqrt{2} = u}$$

$$\Delta = (-2)^2 - 4(1)(-1) = 4 + 4 = 8$$

$$t = \sqrt[3]{x}, \quad u = t^3 \rightarrow t^3 = 1 \pm \sqrt{2}$$

$$t^3 = 1 + \sqrt{2} \rightarrow t = \sqrt[3]{1 + \sqrt{2}}$$

$$t^3 = 1 - \sqrt{2} \rightarrow t = \sqrt[3]{1 - \sqrt{2}}$$

$$\sqrt[3]{x} = \sqrt[3]{1 + \sqrt{2}}, \quad \sqrt[3]{x} = \sqrt[3]{1 - \sqrt{2}} \rightarrow x_1 = 1 + \sqrt{2}, \quad x_2 = 1 - \sqrt{2}$$

$$x_1 + x_2 = (1 + \sqrt{2}) + (1 - \sqrt{2}) = 1 + 1 = \boxed{2}$$

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۶- ریشه‌ها x_1 و x_2

$3x^2 - ax + 4 = 0$

$x_1 = 3x_2$ یا $x_2 = 3x_1$

$S = x_1 + x_2 = \frac{a}{3}$ ، $P = x_1 \cdot x_2 = \frac{4}{3}$

حالت اول: $x_1 = 3x_2 \rightarrow 4x_2 = \frac{a}{3} \rightarrow x_2 = \frac{a}{12}$

$P = 3x_2^2 = \frac{4}{3} \rightarrow x_2^2 = \frac{4}{9} \rightarrow x_2 = \frac{2}{3}$

$\left.\begin{matrix} \frac{a}{12} \\ \frac{2}{3} \end{matrix}\right\} \rightarrow \frac{a}{12} = \frac{2}{3} \rightarrow a = 8$

حالت دوم: $x_2 = 3x_1 \rightarrow S = 4x_1 = \frac{a}{3} \rightarrow x_1 = \frac{a}{12}$

$x_1(3x_1) = 3x_1^2 = \frac{4}{3} \rightarrow x_1 = \frac{2}{3}$

$\rightarrow \frac{a}{12} = \frac{2}{3} \rightarrow a = -8$

چون ترتیب ریشه‌ها عوض شده علامت عوض می‌شود.

$a_1 = 8$ ، $a_2 = -8 \rightarrow a_1 - a_2 = 8 - (-8) = 16$

۷- سهمی رو به بالا

$y = ax^2 + (3 + 2a)x \rightarrow a > 0$

$\alpha = 0$ ، $\beta > 0 \rightarrow S > 0 \rightarrow -\frac{3+2a}{a} > 0 \rightarrow \frac{3+2a}{a} < 0 \rightarrow$ a منفی

۸- $\left.\begin{matrix} y = x^2 + ax - 2 \\ y = -x^2 - 2x + b \end{matrix}\right\}$ راس سهمی‌ها بر هم: $\frac{-a}{2} = \frac{2}{-2} \rightarrow a = 2$

$a = 2$ جایگذاری: $y = x^2 + 2x - 2$ خط $y = 1$ $\rightarrow 1 = x^2 + 2x - 2 \rightarrow (x-1)(x+3) = 0$

$x = 1$

$x = -3$

$-x^2 - 2x + b = 1 \xrightarrow{x=1} -1 - 2 + b = 1 \rightarrow b = 4$

$ab = 2 \times 4 = 8$

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$2x^2 - ax + b = 0$
$2ax^2 + ax - 9 = 0$
$\rightarrow S = \frac{a}{2} = 1 + (-\frac{1}{2}) \rightarrow a = 2 - 1 = 1$ — 9

$a=1 \rightarrow 2x^2 + x - 9 = 0 \rightarrow x^2 + x - 18 \rightarrow (2x+4)(x - \frac{3}{2})$
$\nearrow x = -2$
$\searrow x = \frac{3}{2}$

$x_1 = -2 + \frac{1}{2} = \frac{-3}{2}$
$x_2 = +\frac{3}{2} + \frac{1}{2} = \frac{4}{2} = 2$
$\rightarrow ab = -3 = \frac{b}{2} \rightarrow b = -9$ — 5

$\left[\frac{ab}{4}\right] = \left[\frac{-9}{4}\right] = \left[\frac{-3}{2}\right] = -2$

$x^2 + 9x + m = 0 \rightarrow \alpha_1, \beta_1$
$x^2 + 2x - 3m = 0 \rightarrow \alpha_2, \beta_2$
فرض: $\alpha_1 = \alpha_2 \rightarrow$ ریشه‌های مشترک — 10

$S_1 = \alpha_1 + \beta_1 = -9$
$S_2 = \alpha_2 + \beta_2 = -2$
$(\alpha_2 + \beta_2) - (\alpha_1 + \beta_1) = -2 - (-9) = 4$
$\beta_2 - \beta_1 = 4$

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