به نام خدا

شماره‌ی تکلیف ۲۳ — زهرا اسرابی

$1 = -\frac{1}{2}\cdot 9 - 3 + \frac{17}{2}$

$-\frac{9}{2} - \frac{6}{2} + \frac{17}{2}$

---

۱) $y = ax^2 + bx + c$ ، $A(-1,9)$

$y = a(x - x_s)^2 + y_s \to 1 = a(x+1)^2 + 9 \to 1 = a(3+1)^2 + 9 \to -8 = 16a \to a = -\frac{1}{2}$

$y = -\frac{1}{2}(x+1)^2 + 9 \to -\frac{1}{2}x^2 - x + \frac{17}{2}$

---

۲) $2x^2 + mx + m + 6 = 0$

$\Delta > 0 \to m^2 - 4(2)(m+6) \to m^2 - 8m - 48 > 0 \to (m-12)(m+4) > 0$ ، 12 ، -4

$\frac{c}{a} > 0$ ، $\frac{-b}{a} > 0 \to \frac{-m}{2} > 0 \to m < 0$

$\frac{m+6}{2} > 0 \to m > -6$

$(-6, -4)$

$(-\infty, -4) \cup (12, +\infty)$

---

۳) $3x^2 + (2m-1)x + 2 - m = 0$

$-\frac{b}{a} = \frac{a}{c} \to \frac{1-2m}{3} = \frac{3}{2-m} \to 9 = (1-2m)(2-m) \to 2 - m - 4m + 2m^2 \to 2m^2 - 5m + 2 - 9 = 0 \to 2m^2 - 5m - 7 = 0$

پاسخ: $-\frac{c}{a} = \frac{7}{2}$

(-1) قابل قبول نیست.

$3x^2 - 3x + 3 = 0$

$9 - 4(3)(3) < 0$

---

۴) $x = x^2 - 4$ ، $\alpha = x_1$ ، $\beta = x_2$

$x^2 - x - 4 = 0$ ، $\alpha + \beta = S = -\frac{b}{a} = 1$ ، $\alpha \cdot \beta = P = \frac{c}{a} = -4$

$\beta^3 + \frac{1}{\alpha} + \alpha^3 + \frac{1}{\beta} \to \alpha^3 + \beta^3 + \frac{1}{\alpha} + \frac{1}{\beta}$ ، $\frac{\beta + \alpha}{\alpha\beta} = \frac{1}{-4} = -\frac{1}{4}$

$S^3 - 3SP \to 1 - 3(1)(-4) = 13 - \frac{1}{4} = \frac{51}{4} = S$

$(\beta^3 + \frac{1}{\alpha}) \times (\alpha^3 + \frac{1}{\beta})$

$(\alpha\beta)^3 + \beta^3(\frac{1}{\beta}) + \frac{1}{\alpha}(\alpha^3) + \frac{1}{\alpha\beta} = (\alpha\beta)^3 + \beta^2 + \alpha^2 + \frac{1}{\alpha\beta}$

$(-4)^3 + S^2 - 2P + \frac{1}{-4}$

$P = -64 + 1 - 2(-4) - \frac{1}{4} = -55 - \frac{1}{4} = -\frac{221}{4}$

$x^2 - Sx + P$

$x^2 - (\frac{51}{4})x - \frac{221}{4} \to 4x^2 - 51x - 221 = 0$

---

۵) $\left(\sqrt[3]{x^2} + \frac{1}{\sqrt[3]{x^2}} + 1\right)\left(\sqrt[3]{x^2} - 1\right) = 2\sqrt[3]{x}$ ، t

$x_1 + x_2 = \frac{-b}{a} \to \frac{2}{1} = 2$ پاسخ

$(t^2 + \frac{1}{t^2} + 1)(t^2 - 1) = 2t$

$(\frac{t^4 + 1 + t^2}{t^2})(t^2 - 1) = \frac{t^6 - 1}{t^2} = 2t \to t^6 - 2t^3 - 1 = 0$

$(x^{\frac{1}{3}})^6 - 2(x^{\frac{1}{3}})^3 - 1 = 0 \to x^2 - 2x - 1 = 0$

---

۶) $3x^2 - ax + 4 = 0$

$\alpha = 3\beta$

$\alpha \cdot \beta = \frac{c}{a} = \frac{4}{3} = 3\beta^2 \to 4 = 9\beta^2 \to \frac{4}{9} = \beta^2 \to \beta = \pm\frac{2}{3}$

$\beta = \frac{2}{3} \to \alpha = 2$

$\beta = -\frac{2}{3} \to \alpha = -2$

$\alpha + \beta = \frac{-b}{a}$: $\frac{8}{3} = \frac{a}{3} \to a = 8$ ، $\frac{-8}{3} = \frac{a}{3} \to a = -8$

پاسخ: $|a_1 - a_2| = |8 - (-8)| = 16$

---

۷) $y = ax^2 + (3 + 2a)x$

$a > 0$ ، $\Delta \geq 0$

$(3 + 2a)^2 - 4(a)(0) = 9 + 4a^2 + 12a \geq 0 \to \Delta = 144 - 4(4)(9) = 0$

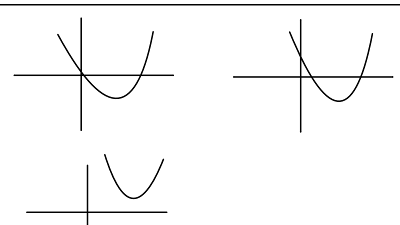
$a = \frac{-b}{2a} = \frac{-12}{8} = -\frac{3}{2}$

$0 \leftarrow \frac{c}{a} \geq 0$

$\frac{-b}{a} > 0$ ، $\frac{-3 - 2a}{a} > 0$

$-\frac{3}{2} < a < 0$

هیچ مقدار a



(۸

$y=-x^{۲}-۲x+b$

$y=x^{۲}+ax-۲$

شرط $x=-\frac{b}{۲a}=\frac{۲}{-۲}=\frac{-a}{۲}\rightarrow ۴=۲a\rightarrow a=۲$

$x^{۲}+۲x-۲=۱\rightarrow x^{۲}+۲x-۳=۰\rightarrow ۱=x$

$\rightarrow -۳=\frac{c}{a}=x$

$-x^{۲}-۲x+b=۱$

$-x^{۲}-۲x+b-۱=۰$

$x=۱\rightarrow -۱-۲+b-۱=۰\rightarrow \boxed{b=۴}$

$x=-۳\rightarrow -۹+۶+b-۱=۰\rightarrow \boxed{b=۴}$

پاسخ $ab=۴\times۲=\boxed{۸}$

(۹

$۲x^{۲}-ax+b=۰$

α,β

$S=-\frac{b}{a}=\frac{+a}{۲}=\alpha+\beta\rightarrow S=\frac{-۱}{۲}$

$P=\frac{c}{a}=\frac{b}{۲}=\alpha.\beta$

$۲ax^{۲}+ax-۶=۰$

$\alpha+\frac{۱}{۲},\ \beta+\frac{۱}{۲}$

$S=\alpha+\beta+۱=\frac{a}{۲}+۱=\frac{-a}{۲}\rightarrow ۱=\frac{-۲a}{۲}\rightarrow \boxed{a=-۱}$

$P=\alpha\beta+\frac{\alpha}{۲}+\frac{\beta}{۲}+\frac{۱}{۴}=\frac{-۶}{۲a}=\frac{-۳}{a}\rightarrow\frac{-۳}{-۱}=۳\rightarrow\frac{b}{۲}-\frac{۱}{۴}+\frac{۱}{۴}=۳\rightarrow\boxed{b=۶}$

$\frac{\alpha+\beta}{۲}\rightarrow\left(\frac{-\frac{۱}{۲}}{\frac{۲}{۱}}\right)=-\frac{۱}{۴}$

$\left[\frac{ab}{۴}\right]=\left[\frac{-۶}{۴}\right]=-۲$

(۱۰

$x^{۲}+۶x+m=۰\ \begin{cases}\alpha\\ \beta\end{cases}\quad \alpha+\beta=-۶$

$x^{۲}+۲x-۳m=۰\ \begin{cases}\alpha\\ \beta'\end{cases}\quad \alpha+\beta'=-۲$

$\alpha+\beta=-۶$

$\times -\quad -\alpha-\beta'=۲$

$\beta-\beta'=-۴$