الف) $y=2x^2-4x+1 \rightarrow a>0 \rightarrow max \rightarrow ext$

$\frac{-b}{2a}=\frac{4}{4}=1$

$\frac{-\Delta}{4a}=\frac{-8}{8}=-1$

ب) $-2x^2+3x-5=y \rightarrow a<0 \rightarrow min \rightarrow ext$

$\frac{-b}{2a}=\frac{3}{4}$

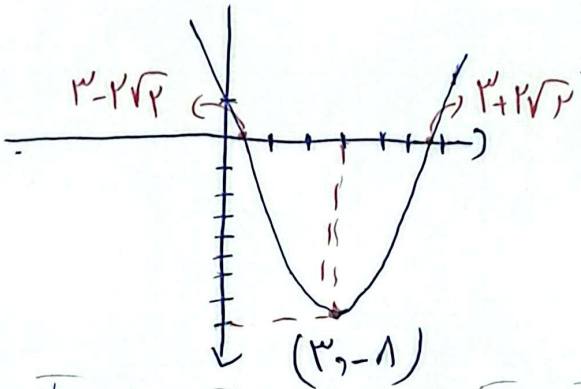
$\frac{-\Delta}{4a}=\frac{31}{-8}$

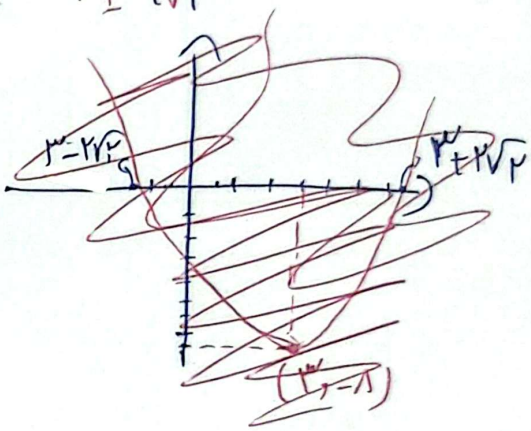
الف) $y=x^2-6x+1$

$ext \mid \frac{-b}{2a}=3$

$9-18+1=-8$

ریشه ها $=\frac{-b\pm\sqrt{\Delta}}{2a}=\frac{6\pm\sqrt{32}}{2}=\frac{6\pm 4\sqrt{2}}{2}=3\pm 2\sqrt{2}$





ب) $y=-x^2+4x+1$

$ext \mid \frac{-b}{2a}=2$

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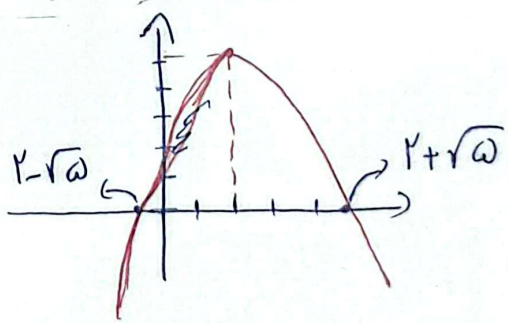
ریشه ها $=\frac{-4\pm\sqrt{16+4}}{-2}=2\pm\sqrt{5}$



$$4(x-\alpha)^2\left(\frac{x}{\alpha}-\beta\right)=4\left(x^2+\alpha^2-2\alpha x\right)(x-\beta)=4\left(x^3-\beta x^2+\alpha^2 x-\beta\alpha^2-2\alpha x^2+2\alpha\beta x\right)$$

$$=4x^3+4(-\beta-2\alpha)x^2-4(\alpha^2+2\alpha\beta x)-4\alpha^2\beta$$

$$-4\alpha^2\beta=-2 \Rightarrow \alpha^2\beta=\frac{1}{2} \rightarrow \alpha\beta(\alpha)=\frac{1}{2}\Rightarrow \alpha=\frac{-1}{4}\Rightarrow \beta=1$$

$$k=4(-\beta-2\alpha)=4\times\frac{12}{2}=\underline{24}$$

$x^2 - 3mx + m = 0 \rightarrow x = \frac{3m \pm \sqrt{9m^2 - 4m}}{2}$

$\Rightarrow \left(\sqrt{\frac{3m + \sqrt{9m^2 - 4m}}{2}} - \sqrt{\frac{3m - \sqrt{9m^2 - 4m}}{2}}\right)^2 = (1)^2$

$\Rightarrow \frac{3m + \sqrt{9m^2 - 4m}}{2} + \frac{3m - \sqrt{9m^2 - 4m}}{2} - 2\sqrt{\frac{9m^2 - 9m^2 + 4m}{4}} = 3m - 2\sqrt{m} = 1$

$\Rightarrow 3m - 2\sqrt{m} - 1 = 0 \rightarrow \sqrt{m} = -\frac{1}{3}$ ✗

$\sqrt{m} = 1$ ✓ $\rightarrow m = 1$

$2x^2 - mx - 4m = 0 \Rightarrow 2x^2 - x - 1 = 0 \rightarrow x = \frac{1 \pm \sqrt{9}}{4}$

$P = \frac{c}{a} = \frac{-1}{2}$

$$\frac{|\alpha-\beta|\times C}{2}=\frac{3}{5}\rightarrow |\alpha-\beta|\times|m|=\frac{3}{2}\rightarrow \frac{\sqrt{4}}{|a|}\times|m|=\frac{3}{2}=\frac{\sqrt{(m+1)^2-4\times 1\times m}}{2}$$

$$\Rightarrow \frac{|m-1|}{2}\times|m|=\frac{3}{2}\rightarrow m^2-2m-3=0\rightarrow m=3,-1$$

$$0<m<3\rightarrow -m^2+2m-3=0\Rightarrow \text{ریشه ندارد}$$

$$y=x^2-mx+1\rightarrow \begin{cases} y=x^2-3x+1\rightarrow \text{طول رأس}\rightarrow \frac{-b}{2a}=\frac{3}{2} \\ y=x^2+x+1\rightarrow \text{طول رأس}=\frac{-b}{2a}=\frac{-1}{2}\end{cases}$$

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$ax^2+bx+a=y \to a>0$

$ax^2+bx+a \geq \frac{V}{8} \Rightarrow$ ~~$ax^2+bx+a-\frac{V}{8}\geq 0$~~

$\to \frac{-\Delta}{4a} = \frac{V}{8} \quad \frac{-9+4a^2}{4a} = \frac{V}{8} \to V_a = -18+8a^2$

$8a^2-Va-18=0 \quad a=4, -\frac{9}{8}$

$\Rightarrow$ ~~$8a^2=36 \Rightarrow a=\pm\sqrt{\frac{36}{8}}$~~

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$x^2-(a+1)x+a=0 \Rightarrow x=\frac{a+1\pm\sqrt{a^2+1+2a-4a}}{2}$ $\to x=a \to a=3 \to P_1 \quad 3$

$\to x=1$

$x^2-(3a+1)x+b=0 \to x^2-10x+b=0 \Rightarrow x=\frac{10\pm\sqrt{100-4b}}{2} \Rightarrow x=5\pm\sqrt{25-b}$

$x_1-x_2=2 \Rightarrow 5+\sqrt{25-b}-(5-\sqrt{25-b})=2 \Rightarrow 2\sqrt{25-b}=2 \Rightarrow \sqrt{25-b}=1 \Rightarrow 25-b=1$

$\to b=24$

$x^2-10x+24=0 \to (x-6)(x-4)=0 \Rightarrow P_2=24$

$P_2-P_1=24-3=21$

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$y = -ax^2 + qx + 2 \Rightarrow ext$ | $\frac{-b}{2a} = \frac{-q}{-2a} = \frac{1}{2}$

$\frac{-1}{4}q + \frac{1}{2}q + 2 \rightarrow \frac{q+8}{4}$

$y = 2bx^2 - bx - 1 \Rightarrow ext$ | $\frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4}$

$\frac{1}{8}b - \frac{1}{4}b - 1 = \frac{-b}{8} - 1$

$\frac{q+8}{4} = 2 \times \frac{1}{4}b - b \times \frac{1}{2} - 1 \rightarrow \frac{q}{4} + 2 = -1 \Rightarrow a = -12$

$\frac{-b}{8} - 1 = -(-12)\left(\frac{1}{16}\right) + 12 \times \frac{1}{4} + 2 \Rightarrow b = -6$

$b - a = -6 - (-12) = 6$

$y = 2\omega\alpha x^2 + 4x + \beta$

$\alpha\beta = \frac{c}{a} = \frac{\beta}{2\omega\alpha} \Rightarrow \alpha = \pm\frac{1}{\omega}$

$a = \frac{1}{\omega} \Rightarrow 2\omega \times \frac{1}{\omega} \times \frac{1}{2\omega} + 4 \times \frac{1}{\omega} + \beta = 0 \Rightarrow \beta = -1 \quad (\alpha \times)$

$a = \frac{-1}{\omega} \Rightarrow 2\omega \times \frac{-1}{\omega} \times \frac{1}{2\omega} + 4 \times \frac{1}{\omega} + \beta = 0 \Rightarrow \beta = 1 \quad (\alpha \checkmark)$

ext | $\frac{-b}{2a} = \frac{-4}{\omega\alpha} = \frac{4}{10}$

$\frac{-4}{4a} = \frac{14-20}{-20} > 0$ ناحیه اول

$x^2 - (a^2 + b^2 - 1^2)x + a + b - 1 = 0 \rightarrow x^2 - Sx + P = 0$

$S = a + b = a^2 + b^2 - 1^2 \longrightarrow ab + 1 = a^2 + b^2 - 1^2 \Rightarrow a^2 + b^2 - ab = 1^2 \Rightarrow a^2 + b^2 + 2ab - 3ab = 1^2$

$P = a \cdot b = a + b - 1 \rightarrow a + b = ab + 1$,

$= 1^2$

$\Rightarrow (a+b)^2 - 3ab = 1^2 \Rightarrow (a+b)^2 - 3(a+b) + 3^2 = 1^2$

$\rightarrow (a+b)(a+b-3) = 10$, $\omega \times 2 = 10 \Rightarrow a + b = \omega$

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