۱-

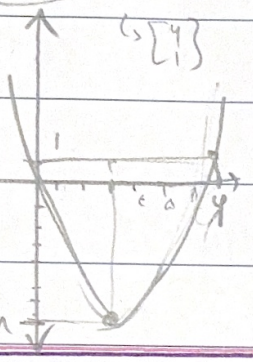
الف) $y = 2x^2 - 4x + 1 \Rightarrow ext = min = \left[\frac{-b}{2a} = 1 \quad ,\ -1\right]$ ← $a > 0$ (به جا نظارت)

ب) $y = -2x^2 + 3x - 5 \Rightarrow ext = max = \left[\frac{b}{2a} = \frac{3}{4} \quad ,\ \frac{-31}{8}\right]$ ← $a < 0$

۲-

الف) $y = x^2 - 4x + 1 \xrightarrow{\sigma_1} [3, -1]$, $y = 1 \Rightarrow x = 0, 4$ ($\sigma_1 \to [4, 1]$)

min



ب) $y = -x^2 + 4x + 1 = y_1$

$[\text{max} = -\frac{b}{2a} = 2]$

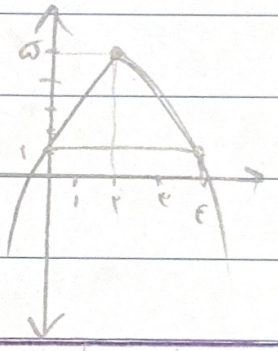
$y = 1$

$-x^2 + 4x = 0$

$x(4 - x) = 0$

$x = 0$

$4 - x = 0 \Rightarrow x = 4$ $[4]$



۳-

$\alpha\beta = -2,\ \alpha + \beta = 1 \Rightarrow$ معادله: $x^2 - x - 2 = 0 \left[\begin{matrix} x = 2 = \alpha \\ x = -1 = \beta \end{matrix}\right]$

ضابطه‌ای $\Rightarrow 4(2)^3 + k(2)^2 - 9(2) - 2 = 0 \Rightarrow 4k = 12 \Rightarrow k = 3$

۴-

$x^2 - 2mx + m = 0 \Rightarrow \alpha + \beta = 2m,\ \alpha\beta = m$

$\sqrt{\alpha} - \sqrt{\beta} = 1$ (به توان ۲) $\Rightarrow (\sqrt{\alpha} - \sqrt{\beta})^2 = 1 \Rightarrow \alpha + \beta - 2\sqrt{\alpha\beta} = 1$ ← ضابطه

$2m - 2\sqrt{m} = 1 \Rightarrow 2m - 2\sqrt{m} - 1 = 0 \longrightarrow \sqrt{m} = t$

$2t^2 - 2t - 1 = 0 \Rightarrow (2t + 1)(t - 1) = 0 \Rightarrow t = 1$ ✓ $\Rightarrow \sqrt{m} = 1 \Rightarrow m = 1$

$t = -\frac{1}{2}$ ✗ $\Rightarrow$ غ ق ق $\sqrt{m} = -\frac{1}{2} \Rightarrow m = \frac{1}{4}$

$2x^2 - x - 1 = 0 \Rightarrow \alpha\beta = \frac{c}{a} = \frac{-1}{2}$

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$|m-2| = \sqrt{(m-2)^2}$

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۵- عبور تابع درجه دوم از مبدأ ⟸ $y=0$

$x_1, x_2 = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{m+2 \pm \sqrt{m^2-4m+4}}{2}$

$x_1 = 1 \Rightarrow x_2 = \frac{m}{2}$ ← طول نقاط $(1,0), (\frac{m}{2},0), (0,m)$

$\frac{1}{2} \times |1-\frac{m}{2}| \times |m| = \frac{m}{4}$ { $|1-\frac{m}{2}|$: قاعده مثلث ، $|m|$: ارتفاع مثلث }

$|m(1-\frac{m}{2})| = \frac{m}{2} \Rightarrow |2m-m^2| = m$ طرفین

$2m-m^2-m=0 \Rightarrow \Delta<0$ ← جواب ندارد

$m^2-2m-m=0$ (قابل قبول)

$(m-3)(m+1)=0$ { $m=3$ ، $m=-1$

$y = x^2-3x+1$ ، $y = x^2+x+1 \Rightarrow -\frac{b}{2a} = \frac{-1}{2}$

$-\frac{b}{2a} = \frac{3}{2}$

۶- نقطه مینیمم

$a>0 \Rightarrow$ min دارد ⟶ $\min_y = \frac{-\Delta}{4a} = \frac{4a^2-9}{4a}$

$\frac{4a^2-9}{4a} = \frac{7}{8} \Rightarrow 8a^2-7a-18=0 \Rightarrow \Delta = 49+576 \Rightarrow a_{1,2} = \frac{7 \pm 25}{16}$

$a_1 = -\frac{9}{8}$ ، $a_2 = 2$ ← غ ق ق با توجه به شرط اول

۷- $2n+1, 2n+3$ ⟹ دو عدد فرد متوالی

$S = 4n+4 = a+1 \Rightarrow a = 4n+3 \Rightarrow n=2a+1, a=3$

$P = (2n+1)(2n+3) = 4n^2+8n+3 = a \Rightarrow 4n^2+8n+3 = 4n+3 \Rightarrow 4n^2+4n=0$

$4n(n+1)=0 \Rightarrow n_1=0$ ✓ ، $n_2=-1$ ← غ ق ق چون طبیعی نیست

معادله $x^2-4x+3=0 \Rightarrow P_1 = \frac{c}{a} = 3$

معادله $x^2-10x+b$

$2m, 2m+2 \Rightarrow \beta = 4m+2 = 10 \Rightarrow m=2 \Rightarrow$ دو عدد: $4, 6 \Rightarrow P_2 = 24$

$P_2 - P_1 = 24-3 = 21$

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-۸ $y = -ax^2 + a + 2 \xrightarrow{\sigma_y} \left(\frac{1}{4}, \frac{a}{4} + 2\right) = ext$

[مثبت] $\leftarrow b - a = -4 - (+12) = 4$

$2b\left(\frac{1}{4}\right)^2 - b\left(\frac{1}{4}\right) - 1 = -1 \Rightarrow \frac{a}{4} + 2 \Rightarrow \frac{a}{4} = -2 \Rightarrow a = -12$

$y = 2bx^2 - bx - 1 \Rightarrow ext = \begin{bmatrix} \frac{b}{4a} = \frac{1}{4} \\ -\frac{b}{8} - 1 \end{bmatrix}$ مثبت $\Rightarrow +b\left(\frac{1}{4}\right)^2 + 12\left(\frac{1}{4}\right) + 2 = \frac{-b}{8} - 1$

$2b = -12 \Rightarrow b = -4$ $\quad \frac{-b}{8} - 1 \quad b = -\frac{4}{2}$

-۹ α مثبت $\Rightarrow \beta = \frac{-2}{4a\alpha} - \frac{1}{4a\alpha} = \frac{-3}{4a\alpha} = \frac{-1}{3a\alpha}$

$\alpha + \beta = \frac{-2}{4a\alpha}$, $\alpha\beta = \frac{\beta}{4a\alpha} \Rightarrow 4a\alpha\alpha\beta - \beta = 0 \Rightarrow$

$\beta(4a\alpha\alpha - 1) = 0 \Rightarrow$ چون β ریشه است پس $\neq 0$

$\hookrightarrow 4a\alpha\alpha - 1 = 0 \Rightarrow$ پس ریشه ها غیر برابرند

$\alpha = \frac{1}{4a\alpha}$

$\hookrightarrow \beta > \alpha \Rightarrow \frac{-1}{3a\alpha} > \frac{1}{4a\alpha}$

$\hookrightarrow -3a\alpha > \alpha \Rightarrow 4a\alpha < 0 \Rightarrow a < 0$

برای کمتر $= \begin{cases} \frac{-b}{2a} = \frac{-2}{4a\alpha} \Rightarrow a\alpha < 0 \Rightarrow \frac{-2}{4a\alpha} > 0 \\ \frac{-\Delta}{4 \times 4a\alpha} \Rightarrow a\alpha < 0 \Rightarrow \frac{-\Delta}{4 \times 4a\alpha} > 0 \end{cases} \Rightarrow$ هر دو مثبت یا هر دو صفر یا هر دو منفی

-۱۰ $a + b = a^2 + b^2 - 12 = (a+b)^2 - 2ab - 12 = S^2 - 2P - 12$

$ab = a + b - 1 \Rightarrow P = S - 1$ $\quad \Rightarrow S = 4$ ✓

$\hookrightarrow S = S^2 - 2(S-1) - 12 \Rightarrow S^2 - 3S - 10 = 0 \Rightarrow (S-5)(S+2) = 0$

$\hookrightarrow S = -2$ ✗

$a + b = S = 5$ $\Leftarrow$ مجموع ریشه‌ها

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۴۳- راه حل دیگر سوال ۴

$S=1, P=-2$

$\begin{cases} 4\alpha^3+k\alpha^2-9\alpha-20 \\ 4\beta^3+k\beta^2-9\beta-20 \end{cases} \xrightarrow{+} 4(\alpha^3+\beta^3)+k(\alpha^2+\beta^2)-9(\alpha+\beta)-40$

$4(S^3-3SP)+k(S^2-2P)-9(S)-40$

$4(1-(-6))+k(1-(-4))-9-40 \Rightarrow 9k=-18 \Rightarrow k=-2$

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