۱

الف) $y=2x^2-4x+1$ $\quad a>0 \to Min$

$Min \left| \begin{array}{l} -\frac{b}{2a}=-\frac{-4}{4}=+1 \\ -\frac{\Delta}{4a}=-1 \end{array}\right. \qquad Min \left| \begin{array}{l} +1 \\ -1 \end{array}\right.$

ب) $y=-2x^2+3x-5$ $\quad a<0 \to Max$

$-2\times\frac{9}{16}+\frac{9}{4}$

$Max \left| \begin{array}{l} -\frac{b}{2a}=-\frac{3}{-4}=+\frac{3}{4} \\ -\frac{\Delta}{4a}=-\frac{9+40}{-8}=+\frac{49}{8} \end{array}\right.$

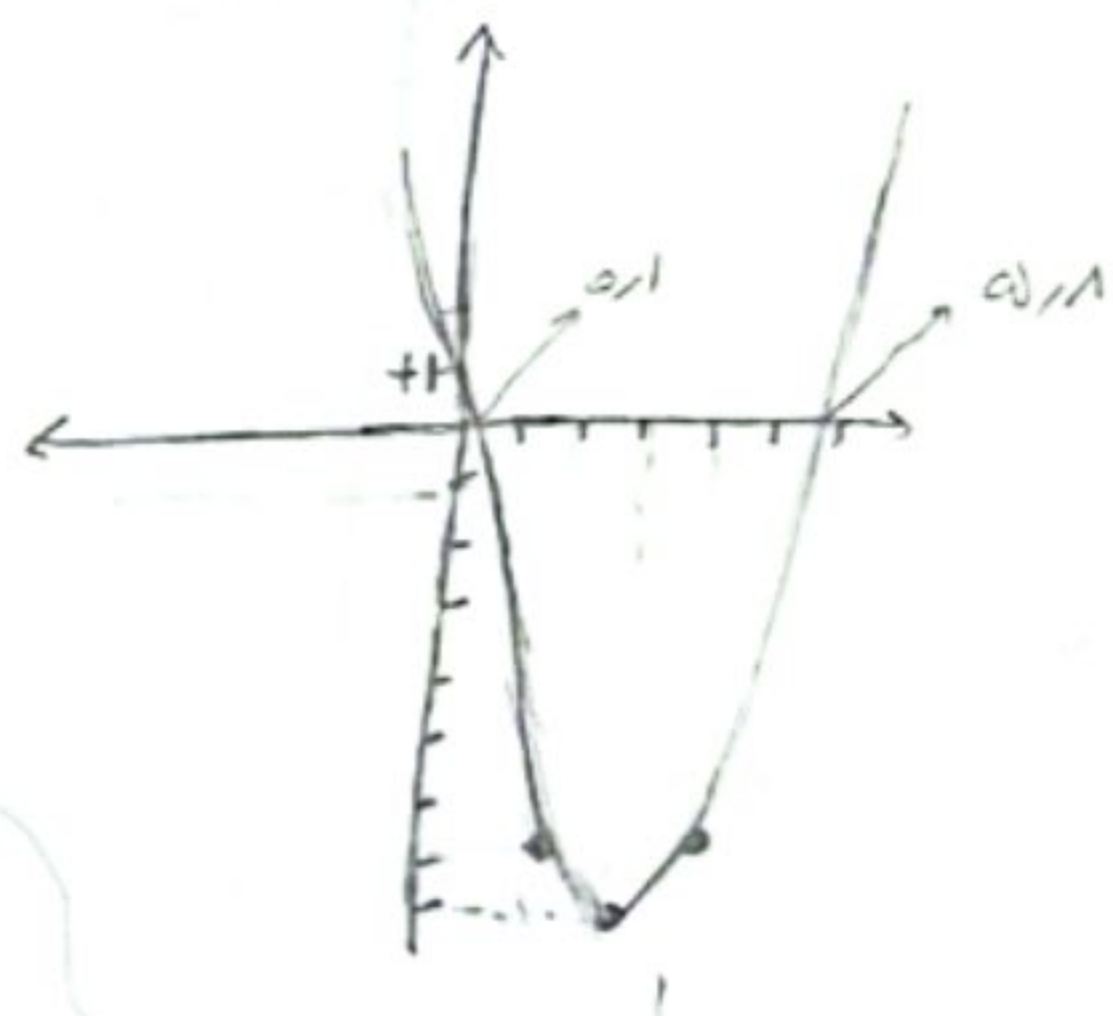
۲

الف) $y=x^2-6x+1$ محور طول‌ها را دو جا قطع می‌کند $\Rightarrow \frac{+6\pm\sqrt{32}}{2} \simeq 5/8$, $\simeq 0/1$

$S \left| \begin{array}{l} -\frac{-6}{2}=+3 \\ -8 \end{array}\right.$

2	3	4
-7	-8	-7

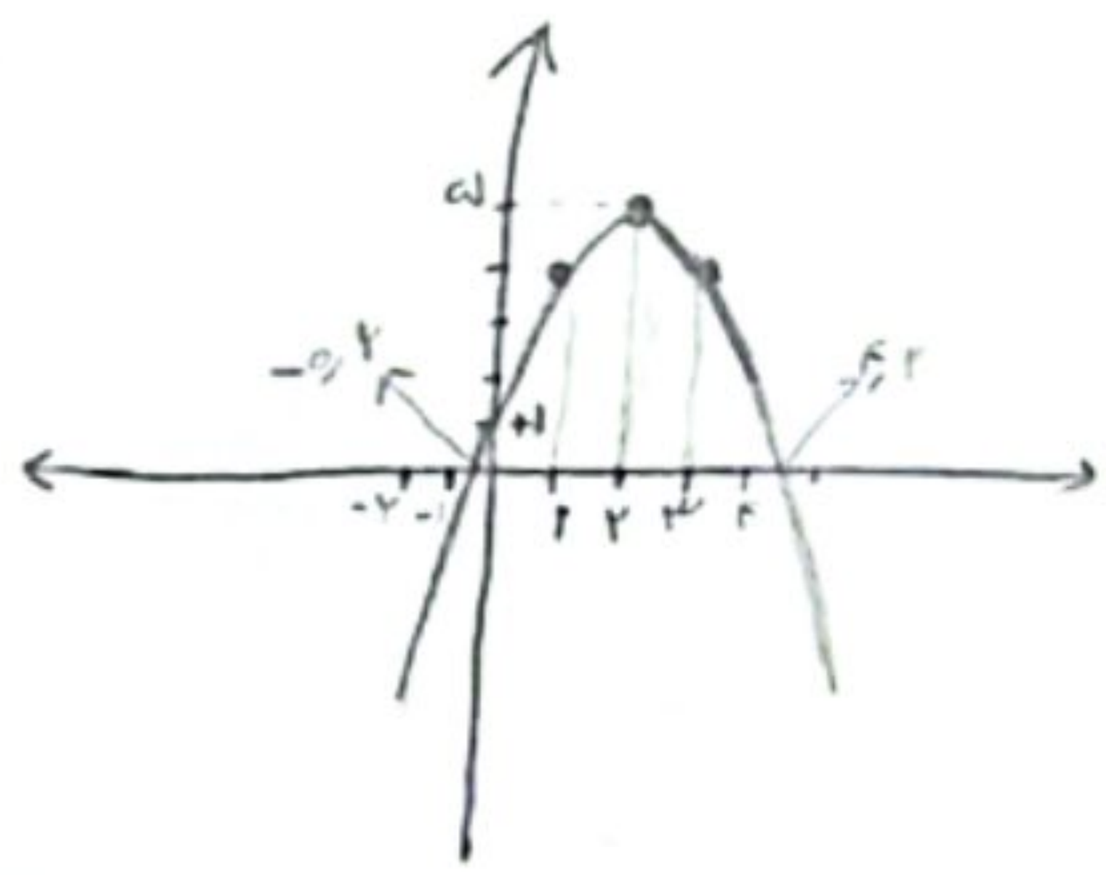
$a>0 \to Min$



ب) $y=-x^2+4x+1$ محور x ها را در دو جا قطع می‌کند $\Rightarrow \frac{-4\pm\sqrt{20}}{-2} \simeq -0/2$, $\simeq 4/2$

$S \left| \begin{array}{l} -\frac{4}{-2}=+2 \\ 5 \end{array}\right.$

1	2	3
4	5	4



۳

$4x^3+kx^2-9x-2=0$

$\alpha+\beta=1$

$\alpha\beta=-2$

ریشه سوم $\to \theta$

$\alpha+\beta+\theta=-\frac{k}{4}$

$\alpha\beta\theta=\frac{2}{4}$, $\alpha\beta=-2 \to \theta=-\frac{1}{4}$

$1-\frac{1}{4}=-\frac{k}{4}$, $+\frac{3}{4}$ $\quad \boxed{k \to -3}$

۴

$x^2-3mx+m=0$

$\alpha+\beta=3m \qquad x^2-Sx+P=0$

$\alpha\beta=m$

$|\sqrt{\alpha}-\sqrt{\beta}|=1 \to (\sqrt{\alpha}-\sqrt{\beta})^2=\alpha+\beta-2\sqrt{\alpha\beta}=1$

$\boxed{t=\sqrt{m}} \qquad 3m-2\sqrt{m}=1$

$m=t^2 \qquad 3m-2\sqrt{m}-1=0$

$3t^2-2t-1=0$

t^2-2t-3 (1×3)

$(t+1)(t-3)$ $\to -\frac{1}{3}$, $\frac{3}{3}=1$

$t=\sqrt{m}\geq 0$ $\quad \boxed{t=1}$

$2x^2-mx-m=0$

$\alpha\beta=\frac{c}{a}=\frac{-m}{2} \Rightarrow \boxed{-\frac{1}{2}}$

(۵)

$y = 2x^2 - (m+2)x + m = 0$

$a+b+c = 0 \rightarrow 1$

$\rightarrow \frac{m}{1}$

$x_1 + x_2 = \frac{m+2}{2}$

$x_1 x_2 = \frac{m}{2}$ $\quad (x_1 > 0)(x_2 > 0)$

$\frac{1}{2} \times$ مساحت مثلث $= \frac{1}{2} \times \frac{|m-2|}{2} \times |m| = \frac{2}{4}$

$|m| \leftarrow (0, m)$

$|m-2| \times |m| = 2$

$|x_1 - x_2| = \frac{\sqrt{\Delta}}{a} \Rightarrow (m+2)^2 - 8m$

$m^2 + 4m + 4 - 8m$

$-4m$

$= (m-2)^2$

$\Delta > 0 \rightarrow$ دو ریشه

$(m-2)^2 > 0 \rightarrow m \neq 2$

$|m-2| \times |m| = 2$ $\quad m = 2$

$m = -1$

$|x_1 - x_2| = \frac{|m-2|}{2}$

$y = x^2 - mx + 1$

$x = \frac{m}{2} \Rightarrow m = \frac{2}{2}$

$m = \frac{-1}{2}$

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(۶)

$y = ax^2 + 2x + a$

$a > 0$

کمترین مقدار عرض $\rightarrow -\frac{\Delta}{4a} \Rightarrow \left(-\frac{9 - 4a^2}{4a} = \frac{7}{8}\right) \Rightarrow -2(9 - 4a^2) = 7a$

$-18 + 8a^2 = 7a$

$8a^2 - 7a - 18 = 0 \rightarrow a^2 - 7a - \frac{144}{19 \times 9}$

$8 \times 18 = 144$

$(a-16)(a+9)$

$\frac{16}{8} = 2$ $\quad -\frac{9}{8}$

تعیین علامت با $\boxed{1}$ مقدار a

$a > 0 \rightarrow \boxed{a = 2}$

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(۷)

$x^2 - (a+1)x + a = 0$ دو عدد فرد متوالی $\rightarrow 2n+1 \quad 2n+3$

$a+1 = 5 \Rightarrow (2n+1) + (2n+3) = 4n+4$ $\quad a+1 = 4n+4 \Rightarrow a = 4n+3$

$a = 5$ $\quad (2n+1)(2n+3) = 4n^2 + 8n + 3$ $\quad 4n^2 + 8n + 3 = 4n + 3 \rightarrow 4n^2 + 4n = 0$

$-1 \quad -4+3 = -1$

$0 \quad -4(0) + 3 = 3$ ✓

$4n(n+1) = 0$

$0, -1$

$\boxed{1 \text{ و } 3}$ $\leftarrow$ پس ۲ عدد فرد

$x^2 - (4b+1)x + b = 0$ دو عدد زوج متوالی $\quad 4m^2 \quad 4m+2$

$a = 2$ $\quad 2a+1 = 5 \Rightarrow 5 \Rightarrow 2m + 4m + 2 = 4m + 2$ $\quad 4m + 2 = 5$ $\quad 4m = 3$ $\boxed{m = 2}$

$b = 2$ $\quad 4 \times 6 = \boxed{24}$

اختلاف $\rightarrow 24 - 2 = \boxed{21}$

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(۸)

$y = -ax^2 + 8x + 2$

$x_{\text{رأس}} = -\frac{b}{2a} \Rightarrow -\frac{8}{-2x} = +\frac{1}{2}$ $\quad y = -a(\frac{1}{2})^2 + a\frac{1}{2} + 2$ $\quad (\frac{1}{2}, \frac{a}{4} + 2)$

$y = 2bx^2 - bx - 1 \rightarrow 2b(\frac{1}{2})^2 - b(\frac{1}{2}) - 1 = \frac{b}{2} - \frac{b}{2} - 1 = -1$

$x_{\text{رأس}} = -\frac{-b}{4b} = \frac{b}{4b} = \frac{1}{4}$ $\quad y = 2b(\frac{1}{4})^2 - b(\frac{1}{4}) - 1 = -\frac{b}{8} - 1$

$\frac{b}{16}$ $\quad \frac{b}{4}$

$y = -(-12)x^2 + (-12)x + 2 = 12x^2 - 12x + 2$

$\frac{1}{4}$

$\frac{9}{4} + 2 = -1$ $\quad \frac{a}{4} = -3$ $\quad -12$

$(\frac{1}{12}, -\frac{b}{8} - 1)$

$\frac{11}{16} - 3 + 2 = \frac{3}{4} - \frac{1}{4} = -\frac{1}{4}$

$-\frac{b}{8} - 1 = -\frac{1}{4}$ $\quad \boxed{b = -6}$

$b - a = -6 - (-12) = \boxed{6}$

$y = 2x^2 + 5x + \beta$

$\left(\alpha\beta = \frac{\beta}{2a}\right) \Rightarrow 2a\alpha^2\beta = \beta$

$2a\alpha^2 = 1$

$\alpha^2 = \frac{1}{2a} \quad \alpha = \pm\frac{1}{a}$

$\alpha = \frac{1}{a} \rightarrow \alpha+\beta = -\frac{5}{2a\alpha} = -\frac{5}{a}$

$\frac{1}{a} + \beta = -\frac{5}{a} \quad \beta = -\frac{1}{\alpha} \quad \beta > \alpha$

$\alpha = -\frac{1}{a} \quad \alpha+\beta = -\frac{5}{2a\alpha} = +\frac{5}{a}$

$-\frac{1}{a} + \beta = \frac{5}{a} \quad \beta = 1 \leftarrow \beta > \alpha$

$y = -\frac{1}{a}x^2 + 5x + 1$

$S \begin{cases} -\frac{b}{2a} = -\frac{5}{2(-a)} = \frac{2}{a} \\ \frac{9}{a} \end{cases}$

(10)

$x^2 - (a^2+b^2-12)x + (a+b-1) = 0$

$a+b = a^2+b^2-12 \Rightarrow S = S^2 - 2P - 12 \Rightarrow S = S^2 - 2S + 2 - 12 \rightarrow S^2 - S - 10 = 0$

$(a+b)^2 = a^2+b^2+2ab$

$ab = a+b-1 \rightarrow P = S-1$

$(S-5)(S+2) = 0$

$S = 5 \quad S = -2$

a و b ← اعداد طبیعی اند