هلیا عرب — ۲۰

۱) الف) $y = 2x^2 - 4x + 1 \rightarrow ext \begin{cases} \frac{-b}{2a} = \frac{4}{4} = 1 \\ \frac{-\Delta}{4a} = -1 \end{cases}$ طول؟ min $\Big|\begin{matrix}1\\-1\end{matrix}$

$a > 0$ min

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ب) $y = -2x^2 + 3x - 5$ max $\Big| \frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4}$, $\frac{-\Delta}{4a} = 9 - 4(-2)(-5) = \frac{-31}{8}$ max $\Big|\begin{matrix}\frac{3}{4}\\ \frac{-31}{8}\end{matrix}$

$a < 0$ max

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۲) $y = x^2 - 6x + 1$ $\quad a > 0$ min $\quad ext \Big| \frac{-b}{2a} = \frac{6}{2} = 3$, $\frac{-\Delta}{4a} = 9 - 18 + 1 = -8$ (عرض رأس)

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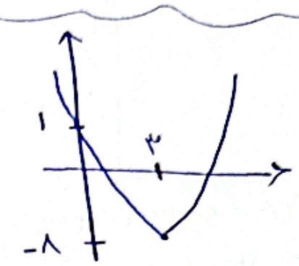
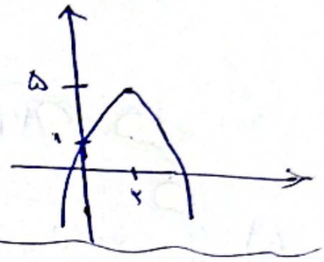


ب) $y = -x^2 + 4x + 1$ $\quad ext \Big| \frac{-b}{2a} = 2$, $\frac{-\Delta}{4a} = -4 + 8 + 1 = 5$ (عرض رأس)

$a < 0 \rightarrow$ max



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۳) $\alpha + \beta = 1 \rightarrow S$

$\alpha \cdot \beta = -2 \rightarrow P$

$x^2 - S + P = x^2 - x - 2$

$(x^2 - x - 2)(\quad) = 4x^3 + 4x^2 - 9x - 2$

$\rightarrow 4x + a \rightarrow 4x^3 + x^2(-4 + a) + x(-8 - a) - 2a$

$4x^3 + 4x^2 - 9x - 2$

$-4 + a = -3$ $\quad -2a = -2$

$a = 1$

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۴) $(\sqrt{\alpha} - \sqrt{\beta})^2 = (1)^2 \Rightarrow \alpha + \beta - 2\sqrt{\alpha \cdot \beta} = S - 2\sqrt{P}$

$3m - 2\sqrt{m} = 1$

$3m - 2\sqrt{m} - 1 = 0 \quad \sqrt{m} = t$

$3t^2 - 2t - 1 = 0$

$\Delta = 4 - 4(3)(-1) = 16$

$t = \frac{2 \pm 4}{6}$ $\nearrow 1 \rightarrow \sqrt{1} = 1$ ✓ $\searrow -\frac{1}{3} \rightarrow \sqrt{-\frac{1}{3}}$ غ ق ق

$3x - x - 1 = 0$

$x = 1 \quad \frac{c}{a} = -\frac{1}{3}$

$-\frac{1}{3} \times 1 = -\frac{1}{3}$

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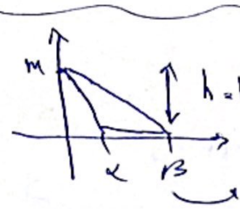
۵) $y = 2x^2 - (m + 2)x + m$



قاعده $= |\alpha - \beta|$ $\quad |\alpha - \beta| \times |m| = 3$

$\frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{m^2 - 4}}{2} = \frac{|m - 2|}{2}$

$|m| \times \frac{|m - 2|}{2} = \frac{3}{2}$

$m \times (m - 2) = 3 \Rightarrow m^2 - 2m = 3$

$m^2 - 2m - 3 = 0$

$(m + 1)(m - 3) = 0$

$m = -1 \quad m = 3$

$\frac{-b}{2a} = -\frac{m}{2} \rightarrow -\frac{1}{2}, \quad \frac{3}{2}$

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۶) $y = ax^2 + 3x + a$

ext | $\frac{-b}{2a} = \frac{-3}{2a}$

$\frac{-\Delta}{4a}$ = عرض رأس → $-\frac{(b^2-4ac)}{4a} = \frac{-9+4a^2}{4a} = \frac{7}{8}$

$28a = -72 + 32a^2 \div 8 \Rightarrow 8a^2 - 7a - 18 = 0 \Rightarrow a^2 - 7a - 144 = 0$

$(a+9)(a-16) = 0$

$a = \frac{-9}{8}$ = غیرق → $a > 0$ min

$a = \frac{16}{8}$ (۲)

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۷) $x^2 - (a+1)x + a = 0$ → اعداد فرد متوالی ⇒ $2k+1$, $2k-1$ → $S = \frac{-b}{a}$ ⇒ $(a+1) = 2k+1+2k-1 = a+1$

$IN \in k$

$a = 4k-1$

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$P = \frac{c}{a} = a \Rightarrow (2k+1)(2k-1) = a$

$4k(k-1)$ ← $4k^2-1 = 4k-1$ ← $4k^2-1 = a$

$k = 0$, $k = 1$ ✓ → $a = 4k-1 = a = 3$

غیرقق ← چون عدد طبیعی اعداد N نیست

$x^2 - (3a+1) + b = 0$ ↗ z ↘ $z+2$

$3a+1 \Rightarrow z + z + 2 = 3a+1 \Rightarrow 2z = 3a-1$

$a = 3$

$b = P = z(z+2) = b$

$4 \times 6 = 24$

$2z = 8 \Rightarrow z = 4$

اختلاف $= 24 - 3 = 21$

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۸) $y = -ax^2 + ax + 2$ → $\frac{-b}{2a} = \frac{1}{2}$

$y = 2bx^2 - bx - 1 = \frac{-b}{2a} = \frac{b}{4b} = \frac{1}{4}$

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$\frac{1}{4}$ ↗ $-a \times \frac{1}{16} + \frac{a}{4} + 2 = \frac{3a + 32}{16}$

↘ $2b \times \frac{1}{16} - b \times \frac{1}{4} - 1 = \frac{-b-8}{8}$

$\frac{3(-12)+32}{16} = \frac{-b-8}{8} = -4b - 32 - 8$

$b = -6$

$\frac{1}{2}$ ↗ $-a \times \frac{1}{4} + \frac{1}{2}a + 2 = \frac{a+8}{4}$

↘ $2b \times \frac{1}{4} - b \times \frac{1}{2} - 1 = -1$

$\frac{a+8}{4} = -1 \to a + 8 = -4$

$a = -12$

$b - a = -6 - (12) = +6$

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۹) $\alpha \cdot \beta = \frac{\beta}{5\alpha}$ ⇒ $y = -5x^2 + 4x + 1$

ext | $\frac{+4}{10}$

$\frac{16+20}{20} = \frac{36}{20}$ → (نامعلوم)

$\alpha = \frac{1}{5}$

$\alpha + \beta = \frac{-4}{25\alpha}$ ↗ $\frac{1}{5} = \beta = \frac{-4}{5} - \frac{1}{5} = -1$ $\alpha < \beta$

↘ $-\frac{1}{5} = \beta = \frac{4}{5} + \frac{1}{5} = 1$ ✓

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۱۰) $a \cdot b = a + b - 1$ → $P = S - 1$

$a + b = a^2 + b^2 - 12$ → $S = S^2 - 2P - 12$ ⇒ $S = S^2 - 2(S-1) - 12$

$S = S^2 - 2S - 10$

$S^2 - 3S - 10 = 0$

$S = 5$

$S = -2$ $\alpha = a + b$

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