۲۰ آنالیز ریاضی

۱) الف) $y = 2x^2 - 4x + 1$

$a > 0 \rightarrow 2 > 0 \rightarrow \min$

$\Rightarrow ext \begin{cases} \frac{-b}{2a} = \frac{4}{4} = 1 \\ \frac{-\Delta}{4a} = \frac{-(16 - 4 \times 2 \times 1)}{2 \times 4} = -1 \end{cases}$

ب) $y = -2x^2 + 3x - 5$

$a < 0 \rightarrow -2 < 0 \rightarrow \max$

$ext \begin{cases} \frac{-b}{2a} = \frac{-3}{-4} = \frac{3}{4} \\ \frac{-\Delta}{4a} = \frac{-(9 - 4 \times -2 \times -5)}{4 \times -2} = \frac{41}{-8} \end{cases}$

۲) الف) $y = x^2 - 4x + 1$ $\quad a > 0 \rightarrow \min$

$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 1}}{2} = 2 \pm 2\sqrt{2}$

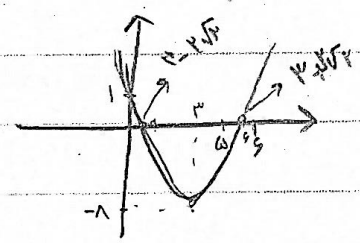
$ext \begin{cases} \frac{-b}{2a} = 2 \\ \frac{-\Delta}{4a} = -4 \end{cases}$

ب) $y = -x^2 + 4x + 1$ $\quad a < 0 \rightarrow \max$

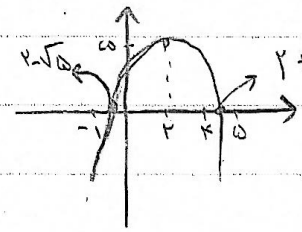
$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-4 \pm \sqrt{16 - 4 \times -1 \times 1}}{-2} = \frac{-2 \pm \sqrt{5}}{-1} = 2 \pm \sqrt{5}$

$ext \begin{cases} \frac{-b}{2a} = 2 \\ \frac{-\Delta}{4a} = 5 \end{cases}$

الف)



ب)



تقسیل ←

۳) $\alpha\beta = -2$ $\quad 4x^3 + kx^2 - 9x - 2 = 0$ $\quad k = ?$

$\alpha + \beta = 1$

$(x^2 - x - 2)(4x + a) = 4x^3 + 4x^2 - 9x - 2$

$\hookrightarrow 4x^3 + x^2(-4 + a) + x(-8 - a) - 2a =$

$4x^3 + 4x^2 - 9x(-2)$

$-4 + a = -3$

$a = +1$

$k = 1$

۴- $x^2 - 3mx + m = 0$ $(\sqrt{m} = t)$

$(\sqrt{\alpha} - \sqrt{\beta})^2 = 1^2 \rightarrow \alpha + \beta - 2\sqrt{\alpha\beta} = S - 2\sqrt{P}$

$3m - 2\sqrt{m} = 1 \rightarrow 3t^2 - 2t - 1 = 0$ $\quad \Delta = 4 - 3 \times 4 + 11 = 16$

$3\alpha - \alpha - 1 = 0$

$\alpha = 1 \quad \frac{c}{a} = \frac{-1}{3}$ $\quad t = \frac{2 \pm 4}{2} < \begin{matrix} 1 & \sqrt{1} & \checkmark \\ -\frac{1}{3} & \rightarrow \sqrt{-\frac{1}{3}} & \times \end{matrix}$

$\frac{-1}{3} \times 1 = \sqrt{\frac{-1}{3}}$

۵- $y = 2x^2 - (m+2)x + m = 0$ $\xrightarrow{a+b+c=0}$

$x = 1,$ $\quad |a-\beta| \times |m|$

$S = \left|m\left(\frac{m}{2} - 1\right)\right| = \frac{3}{2}$ $\quad \hookrightarrow \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{m^2 - 4}}{2} = \frac{|m-2|}{2}$

$m \times (m-2) = 3 \quad m^2 - 2m = 3 \quad (m-1)(m-3) = 0 \quad \frac{-b}{2a} = \frac{-m}{2} = \begin{matrix} -\frac{1}{2} \\ \frac{3}{2} \end{matrix}$

$\hookrightarrow m = -1 \quad \hookrightarrow m = 3$

۶- $y = ax^2 + 2x + a$ $\quad$ ext

$\frac{-b}{2a} = \frac{-2}{2a}$

$\frac{-\Delta}{4a} = \frac{-4 + 4a^2}{4a} = \frac{7}{8}$

$28a = -28 + 32a^2 \div 4 \rightarrow 8a^2 - 7a - 14 = 0 \rightarrow a^2 - 7a + 14 = 0$

$(a+2)(a-7) = 0$

$a = \frac{-2}{4} \times \quad , a = \frac{7}{4} = 2$

۷- $x^2 - (a+1)x + a + 2 = 0$ $\quad S = a+1 \rightarrow P = a$

$2n + 2 = a + 1$

$a = 2n + 1$ $\quad n(n+2) = a$

$n^2 - (2a+1)x + b = 0$ $\quad n^2 + 2n = 2(2n+1) \rightarrow n \pm 1 < \begin{matrix} n = 1 \\ n+2 = 3 \end{matrix}$

$m, m+2$

$m + m + 2 = 10$

$2m + 2 = 10$

$m = 4$

$|\alpha\beta - \alpha_2\beta_2| = |3 - 24| = 21$

$y = ax^2 + ax + 2 \longrightarrow ext \left| \begin{matrix} \frac{1}{2} \\ \frac{a^2 + 8a}{4a} \end{matrix} \right.$

(۲) ـ (۸)

$y = 2bx^2 - bx - 1 \longrightarrow ext \left| \begin{matrix} \frac{1}{4} \\ \frac{b^2 + 8b}{-8b} \end{matrix} \right.$

$b - a = -4 - (-12) \sqrt{4}$

$2b\left(\frac{1}{4}\right) - b\left(\frac{1}{2}\right) - 1 = \frac{a}{4} + 2 \longrightarrow a = -12$

$\frac{-a}{14} + \frac{a}{4} + 2 = \frac{-b}{8} - 1 \longrightarrow \frac{12}{14} = \frac{-b}{8} \longrightarrow b = -4$

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(۲) ـ (۹)

$\alpha\beta = \frac{\beta}{25\alpha} \longrightarrow \alpha^2 = \frac{1}{25}$

$\frac{-1}{5\beta} = \alpha$ $\quad 25\alpha^2 + 25\alpha\beta + 4 = 0$ $\quad ext \left| \begin{matrix} \frac{+4}{10} \\ \frac{14 + 20}{20} = \frac{34}{20} \end{matrix} \right.$ $\quad \longrightarrow$ نامه اول

$25\alpha\beta = -5 - 1$

$a = \frac{-1}{5}$ یا $\frac{1}{5}$ ✗ $\quad 5\beta = -1$

$\beta = \frac{-1}{5}$

$\frac{5}{\beta} = 5\beta^2 \longrightarrow 5\beta^3 - 5 = 0$ $\quad$ (1) ✓ $\quad$ (-1) ✗

$5(\beta^3 - 1) = 0$ $\quad (\beta - 1)(\beta + 1)$

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(۲) ـ (۱۵)

$a \times b = a + 1 - 1 \longrightarrow p = s - 1$

$a + b = a^2 + b^2 - 12 \longrightarrow s = s^2 - 2p - 12$

$s = s^2 - 2(s-1) - 12$

$s = s^2 - 2s - 15$

$s^2 - 3s - 15 = 0 \longrightarrow s = 5$

$s = -2\alpha \longrightarrow a + b$