ریاضی

کوشا بهینی

۲۰

(۱) $y = 2x^2 - 4x + 1$ $\quad ext = \left|\begin{array}{l} 1 \\ 2-4+1=-1 \end{array}\right.$ $\quad y = -2x^2 + 3x - 5$ $\quad ext \left|\begin{array}{l} \frac{3}{4} \\ -\frac{31}{8} \end{array}\right.$

(۲) $\min - \cup$ $\qquad \max - \cap$

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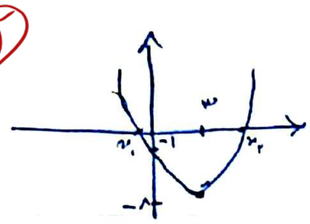
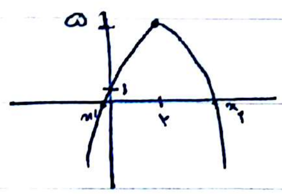
(۲) $y = x^2 - 6x + 1$ $\quad ext = \left|\begin{array}{l} 3 \\ -8 \end{array}\right.$ min $\qquad y = -x^2 + 4x + 1$ $\quad ext_2 \left|\begin{array}{l} 2 \\ 5 \end{array}\right.$ max

(۲)





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(۳) (۲) $\alpha\beta = \frac{c}{a} = -2 \quad , \alpha + \beta = \frac{-b}{a} = 1$

$\alpha + \beta + \gamma = \frac{-K}{4}$

$\alpha\beta\gamma = \frac{2}{4} = \frac{1}{2} \longrightarrow (-2)\gamma = \frac{1}{2} \Rightarrow \gamma = -\frac{1}{4}$ ریشه سوم

$1 + \left(\frac{-1}{4}\right) = \frac{-K}{4} \Rightarrow K = -3$

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(۴) $\alpha\beta = \frac{c}{a} = m \quad , \quad \alpha + \beta = \frac{-b}{a} = 3m \qquad x^2 - 3mx + m = 0$

(۲) $|\sqrt{\alpha} - \sqrt{\beta}| = 1$

به توان ۲ می‌رسانیم $\longrightarrow \underbrace{\alpha + \beta}_{3m} - 2\underbrace{\sqrt{\alpha\beta}}_{m} = 1 \longrightarrow 3m - 2\sqrt{m} = 1$

$\Rightarrow \left[2\sqrt{m} = 3m - 1\right]^2 \longrightarrow 4m = 9m^2 + 1 - 6m \Rightarrow 9m^2 - 10m + 1 = 0$

$\Rightarrow \Delta = 64 \longrightarrow m = \frac{10 \pm 8}{18}$ $\quad m_1 = 1 \xrightarrow{\text{جایگذاری}} \checkmark \ 1 = 1$ $\quad m_2 = \frac{1}{9} \xrightarrow{\text{جایگذاری}} \times \ -\frac{1}{3} \neq 1$

$\Rightarrow 2x^2 - x - 1 = 0 \longrightarrow \alpha\beta = \frac{c}{a} = \frac{-1}{2}$

به نام خدا

کوثر عابدینی

SUBJECT:

Year Month Day Page:( )

(۵) $2x^2-(m+2)x+m$

3

$\xrightarrow{x=1} 2-m-2+m=0$ ✓ $\Rightarrow x_1=1 \Rightarrow (1,0)$ ①

$\alpha\beta=\frac{c}{a}=\frac{m}{2} \Rightarrow 1\times x_2=\frac{m}{2} \Rightarrow x_2=\frac{m}{2} \Rightarrow (\frac{m}{2},0)$ ②

نقطه تقاطع با محور عرض‌ها → $(x=0) \Rightarrow y_s=f(0)=m \Rightarrow (0,m)$ ③

① و ② یک قاعده و ③ محور عرض‌ها

$b_{\text{قاعده}}=|x_1-x_2|=|1-\frac{m}{2}|$, $h_{\text{ارتفاع}}=|m|$

$\frac{1}{2}\times b\times h=\frac{3}{4} \Rightarrow \frac{1}{2}\times|1-\frac{m}{2}|\times|m|=\frac{3}{4} \xrightarrow{\times 4} 2|1-\frac{m}{2}|\times|m|=3$

$\Rightarrow |2-m|\times|m|=3$

1- $m>2 \Rightarrow m^2-2m-3=0 \Rightarrow (m-3)(m+1)=0$ → $m=3$ ✓ , $m=-1$

2- $0<m<2 \Rightarrow 2m-m^2=3 \Rightarrow m^2-2m+3=0$ $\Delta<0$ ×

3- $m<0 \Rightarrow -2m+m^2=3 \Rightarrow m^2-2m-3=0 \Rightarrow (m-3)(m+1)=0$ → $m=3$ × , $m=-1$

$\Rightarrow y=x^2-mx+1$ $x_s=\frac{m}{2} \to m=3 \to x_s=\frac{3}{2}$

$m=-1 \to x_s=-\frac{1}{2}$

(۶) ax^2+3x+a کمترین مقدار $\Rightarrow a>0$

3

$\frac{-\Delta}{4a}=\frac{4a^2-9}{4a}=\frac{7}{8}$

$32a^2-72=28a \Rightarrow 32a^2-28a-72=0 \Rightarrow 8a^2-7a-18=0 \Rightarrow a^2-7a-144=0$

$(a-16)(a+9)=0$ → 16 ✓ , -9 ×

$\Rightarrow a=16$

Celine

۲

۷ ① $x_1 = 2k-1$, $x_2 = 2k+1 \to k \in \{1,2,\dots\}$

$\to x_1 + x_2 = \frac{-b}{a} = a+1 \Rightarrow (2k-1)+(2k+1) = a+1 \Rightarrow 4k = a+1 \Rightarrow a = 4k-1$

$\to x_1 x_2 = \frac{c}{a} = a \Rightarrow (2k-1)(2k+1) = a \Rightarrow 4k^2-1 = a \Rightarrow a = 4k^2-1$

$\Rightarrow 4k-1 = 4k^2-1 \Rightarrow 4k(k-1) = 0 \xrightarrow{k \geq 1} k = 1$

$a = 4(1)-1 = 3$; $x_1 = 2(1)-1 = 1$, $x_2 = 2(1)+1 = 3$; ضرب $p = a = 3$

② $a = 3 \to x^2 - 10x + b = 0$ $x_1 = 2m$, $x_2 = 2m+2 \to m \in \{1,2,\dots\}$

۵ $\to x_1 + x_2 = 10 \Rightarrow (2m) + (2m+2) = 10 \Rightarrow 4m = 8 \Rightarrow \boxed{m=2}$

$\to m = 2 \to x_1 = 4$, $x_2 = 6$

$\Rightarrow p = b = 4 \times 6 = 24$

$\Rightarrow |p_1 - p_2| = |3-24| = \boxed{21}$

۸ $y = -ax^2 + ax + 2$ $y = 2bx^2 - bx - 1$

۳ ent $\left| \begin{matrix} \frac{1}{4} \\ \frac{a+8}{4} \end{matrix} \right.$ $\frac{b}{4} - \frac{b}{2} - 1 = \frac{8+a}{4}$ ent $\left| \begin{matrix} \frac{1}{4} \\ \frac{-b-8}{8} \end{matrix} \right.$

$8 + a = -4 \Rightarrow \boxed{a = -12}$

$\to \frac{-a}{16} + \frac{4a}{16} + \frac{32}{16} = \frac{3a+32}{16}$

$\Rightarrow \frac{-36+32}{16} = \frac{-b-8}{8} \Rightarrow -\frac{1}{4} = -\frac{(b+8)}{8} \Rightarrow \boxed{b = -6}$

$|b-a| = |-6+12| = 6$

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[illegible]

9

$y = 25ax^2 + 4x + B$

$\alpha + \beta = \frac{-4}{25a} \qquad \alpha\beta = \frac{B}{25a} \qquad B \neq 0$

$\alpha = \frac{1}{25\alpha} \Rightarrow 25\alpha^2 = 1 \Rightarrow \alpha^2 = \frac{1}{25} \Rightarrow \alpha = \pm\frac{1}{5}$

$\therefore \beta = \frac{-4}{25\alpha} - \alpha$

$\alpha = -\frac{1}{5}$

$\alpha = \frac{1}{5} \rightarrow \beta = \frac{-4}{25(1/5)} - \frac{1}{5} \Rightarrow \beta = -1 \quad \times$

$\alpha = \frac{1}{5}$

$\alpha = -\frac{1}{5} \rightarrow \beta = \frac{-4}{25(-1/5)} + \frac{1}{5} \Rightarrow \beta = 1 \quad \checkmark \ \beta > \alpha$

$\rightarrow \alpha = -\frac{1}{5}, \ \beta = 1 \qquad 25\alpha = -5 \ (\alpha = -\frac{1}{5}) \qquad a < 0 \Rightarrow \cap$

$x_S = \frac{2}{5} \qquad \frac{2}{5} > 0 \qquad , \ y_S = \frac{9}{5} \qquad \frac{9}{5} > 0 \qquad \Rightarrow$ I ربع

10

$x^2 - (a^2 + b^2 - 12)x + a + b - 1 = 0$

$a + b = -\frac{-(a^2 + b^2 - 12)}{1} = a^2 + b^2 - 12$

$ab = \frac{a + b - 1}{1} = a + b - 1 \Rightarrow ab = a + b - 1 \Rightarrow \boxed{a + b = ab + 1}$

جایگذاری $\rightarrow \ ab + 1 = a^2 + b^2 - 12 \Rightarrow ab + 13 = a^2 + b^2 \Rightarrow ab + 13 = (a + b)^2 - 2ab$

$\Rightarrow 13 = (a + b)^2 - 3\underbrace{ab}_{a+b-1} \Rightarrow 13 = (a + b)^2 - 3(a + b) + 3$

$\Rightarrow 10 = (a + b)^2 - 3(a + b)$

$\xrightarrow{a+b=S} S^2 - 3S - 10 = 0 \Rightarrow (S - 5)(S + 2) = 0 \quad S_1 = -2 \ \times, \quad S_2 = 5$

حقیقی $b, a \Rightarrow a + b = -2 \ \times \ \rightarrow a + b = 5 \ \checkmark$