تمرین اسلایدی

① $y = 2x^2 - 4x + 1$

$x = \frac{-b}{2a} = \frac{-(-4)}{4} = 1$

رأس: $(1, -1)$

اکسترمم: min

$y = \frac{-\Delta}{4a} = \Delta = b^2 - 4ac \rightarrow 16 - 4(2 \times 1) = 8 \rightarrow \frac{-8}{8} = -1$

$y = -2x^2 + 3x - 5$

رأس: $(\frac{3}{4}, -\frac{31}{8})$

اکسترمم: max

$x = \frac{-3}{-4} = \frac{3}{4}$

$y = 9 - 4(-2 \times 5) \rightarrow 9 - 40 = -31 \rightarrow -\frac{31}{8}$

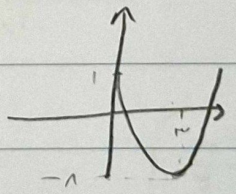
② $y = x^2 - 4x + 1$

رأس: $(2, -3)$

$x = \frac{4}{2} = 2$

جایگذاری

$y = 9 - 12 + 1 = -3$

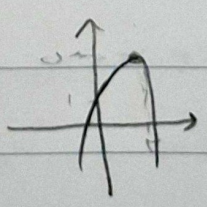


$y = -x^2 + 4x + 1$

رأس: $(2, 5)$

$x = \frac{-4}{-2} = 2$

$y = -4 + 8 + 1 = 5$



③

$4x^3 + 12x^2 - 9x - 3 = 0 \rightarrow x^2 - (\alpha+\beta)x + \alpha\beta$

$x^2 - 1x - 2 \rightarrow 4x^3 + 12x^2 - 9x - 3 \;\Big|\; \frac{x^2 - x - 2}{4x + (k+4)}$

$k = 0$ ⟩

④

$x^2 - 3mx + m = 0$

$|x_1 - x_2| = \frac{\sqrt{\Delta}}{|a|} \rightarrow \Delta = 9m^2 - 4m$

$\sqrt{9m^2 - 4m} = 1 \rightarrow 9m^2 - 4m = 1 \rightarrow 9m^2 - 4m - 1 = 0$

$m = \frac{4 \pm \sqrt{16+36}}{18} = \frac{4 \pm \sqrt{52}}{18} = \frac{2 \pm 2\sqrt{13}}{9}$

حاصل ضرب $\rightarrow -\frac{1}{9}$ ⟩

⑤

$y = 4x^2 - (m+2)x + m$

$\Delta = (m+2)^2 - 16m = m^2 + 4m + 4 - 16m = m^2 - 12m + 4 = (m-6)^2$

$x = \frac{(m+2) \pm (m-6)}{8} \rightarrow x_1 = \frac{2m}{8} = \frac{m}{4}$

$\rightarrow x_2 = \frac{8}{8} = 1$ ⟩

s.a.m

$$\frac{1}{2} \times \left|1 - \frac{m}{2}\right| \times |m| = \frac{3}{2} \quad \begin{cases} \rightarrow m = 1 \\ \rightarrow m = 3 \end{cases}$$

$x = \frac{m}{2}$ اگر $m = 1 \rightarrow x = \frac{1}{2}$

$\rightarrow m = 3 \rightarrow x = \frac{3}{2}$

(۶)

۶. $ax^2 + 2x + a$

$x = \frac{-2}{2a}$

$y_{min} = a - \frac{9}{4a} \rightarrow a - \frac{9}{4a} = \frac{7}{8}$

$8a^2 - 8 = 7a \rightarrow 8a^2 - 7a - 8 = 0 \rightarrow a = \frac{7 \pm 25}{16} \nearrow a = 2$ $\searrow a = -\frac{9}{8}$

(۷)

$x^2 - (a+1)x + a$

ضرب: $n, n+2$

مجموع: $2n + 2 = a + 1$

ضرب: $n(n+2) = a$

$\rightarrow n = 1$

مجموع: $1 + 3$

$a = 3 \rightarrow x^2 - (3-1)x + 3 = 0 \rightarrow x^2 - 2x + 3 = 0$

مجموع ریشه‌ها: $1 \times 3 = 3$

ضرب ریشه‌ها: $\alpha\beta = 3$

⑥ رأس برابر $x_1 = x_2$

$y_1 = ax^2 + bx + 1$

$x_1 = \frac{-b}{2a}$

$y_2 = ax^2 + (b-2)x + 1$

$x_2 = \frac{-(b-2)}{2a}$

$\frac{-b}{2a} = \frac{-(b-2)}{2a}$

$-b = -(b-2)$

$-b = -b + 2$

$0 = 2$

$b - a = 1$

⑦

$y = 25ax^2 + 6x + \beta$

چون $\beta > \alpha$

$+a$ علامت Δ در ① مثبت

مختصات رأس < 0

⑮

$x^2 - (a^2 + b^2 - 13)x + a + b - 1 = 0$

چون ریشه‌ها طبیعی‌اند $\rightarrow$ $a + b = 14$

$\alpha\beta = 12$

s.a.m