دهم دفتر A — حلما عامری — ۱۸ — پاسخنامه تکلیف شماره ۲۵

۱) الف:

$y = 3x^2 - 2x \quad x_s = \frac{-b}{2a} = \frac{2}{6} = \frac{1}{3} \quad y_s = \frac{4ac-b^2}{4a} = \frac{-4}{12} = \frac{-1}{3}$ (min)

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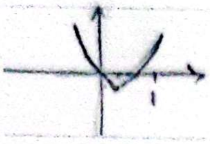
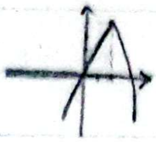


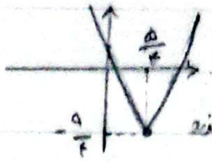
از ناحیه چهارم می‌گذرد

ب: $y = -x^2 + 4x \quad x_s = \frac{-4}{-2} = 2 \quad y_s = -4 + 8 = 4$ (max)



از ناحیه دوم نمی‌گذرد

۲) الف: $y = 2x^2 - 5x + 2 \quad x_s = \frac{5}{4} \quad y_s = \frac{16-25}{8} = \frac{-9}{8}$ (min)



از ناحیه ۱ و ۲ و ۴ می‌گذرد

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ب: $y = -x^2 + 4x - 1 \to x_s = \frac{-4}{-2} = 2 \quad y_s = -4 + 8 - 1 = 3$ (max)

از ناحیه ۱ و ۳ و ۴ می‌گذرد

۳) الف: $\frac{\frac{-b}{a}}{\frac{\sqrt{\Delta}}{|a|}} = \frac{\frac{1}{1}}{\frac{\sqrt{1+12}}{1}} = \frac{1}{\sqrt{13}}$

ب: $S^2 - 2P = (-\frac{b}{a})^2 - 2(\frac{c}{a}) = (\frac{1}{1})^2 - 2(\frac{-3}{1}) = 7$

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ج: $\alpha^3 + \beta^3 = S^3 - 3SP = (\frac{1}{1})^3 - 3(1)(-3) = 10$

د: $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta) = (\sqrt{13})(7-3) = 4\sqrt{13}$

۴) ۳

$x = 2 \quad \Delta = 0 \quad \Delta < 0 \to a^2 - 4a < 0 \to a(a-4) < 0 \to 0 < a < 4$

$\hookrightarrow 2^2 - 2a + a = 0 \to a = 4 \Rightarrow a \in (0, 4]$

۵) ۳

$\alpha + \beta = \frac{12}{3} = 4 \quad \alpha . \beta = \frac{-a}{3} \quad 2\alpha^2 + (4-\alpha)^2 - 4\alpha - 7 = 0$

$\hookrightarrow \beta = 4 - \alpha \quad 3\alpha^2 - 12\alpha + 9 = 0 \quad \alpha^2 - 12\alpha + 27 = 0 \quad (\underbrace{\alpha - 3}_{3})(\underbrace{\alpha - 9}_{9}) = 0$

$\frac{3}{3} = 1$ و $\frac{9}{3} = 3$

$\Rightarrow \alpha . \beta = 3 \to a = -9 \to \frac{-9}{3} = -3$ برابر

(1) $\beta = 3$ (2) $\beta = 1$

۶) ۳

$b = \frac{(2a+3) + (7-2a)}{2} = 5 \to S = 5, 3 \quad a - 2 \geq 1 \Rightarrow a \geq 3 \quad 7 - 2a \geq 1 \quad a \leq 3$

$\Rightarrow a = 3 \Rightarrow A = (1, 1) \to y = a(x-5)^2 + 3 \xrightarrow[(9,1)]{\text{جایگذاری}} 1 = a(9-5)^2 + 3 \to a = \frac{-1}{8}$

$\Rightarrow y = -\frac{1}{8}(x-5)^2 + 3 \quad x = 0 \to y = \frac{-25}{8} + \frac{24}{8} = \frac{-1}{8} = (0, -\frac{1}{8})$ نقطه برخورد

$d = \sqrt{0^2 + (-\frac{1}{8})^2} = \frac{1}{8}$

سوال ۸

$x_s = \frac{-5+1}{2} = -2$

$f(x) = a(x+2)^2 - \frac{1}{2} \to \frac{3}{2} = a(0+2)^2 - \frac{1}{2} \to a = \frac{1}{2}$

$(1, \beta) \in f(x) \to \beta = \frac{1}{2}(1+2)^2 - \frac{1}{2} \to \beta = 4$

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$\alpha+\beta=1 \qquad \beta=1-\alpha \qquad 40(1-\alpha)^2+2\alpha^2-20(1-\alpha)=17$

$40\alpha^2-40\alpha+3=0 \rightarrow S=1\,,\ P=\frac{1}{20} \qquad |\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\frac{\sqrt{\frac{4}{5}}}{1}=\boxed{\frac{2}{\sqrt{5}}}$ ۳

$\rightarrow \alpha^2-\alpha+\frac{1}{20}=0$

راه حل درست صفحه قبل

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~~$\left(\frac{3}{2}\right)=k(0-3)^2-\frac{1}{2} \rightarrow \frac{3}{2}=9k-\frac{1}{2} \Rightarrow k=\frac{2}{9} \Rightarrow y=\frac{2}{9}(x-3)^2-\frac{1}{2}$~~ ۵

$\beta=\frac{2}{9}(1-3)^2-\frac{1}{2}=\boxed{\frac{17}{18}}$

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$x^2+6x+a=0 \begin{cases} \alpha=-3+\sqrt{9-a} \rightarrow \alpha^2=18-a-6\sqrt{9-a} \\ \beta=-3-\sqrt{9-a} \rightarrow \beta^2=18-a+6\sqrt{9-a} \end{cases}$ ۳

$3\alpha^2+2\beta^2=90-5a-6\sqrt{9-a}=12\sqrt{2}+85 \rightarrow 5a+6\sqrt{9-a}=5+6\sqrt{8}$

$\left.\begin{array}{l} 90-5a+6\sqrt{9-a}=12\sqrt{2}+85 \\ 5-5a+6\sqrt{9-a}=12\sqrt{2} \end{array}\right\} \ \cancel{6}_{1}\sqrt{9-a}=\cancel{12}_{2}\sqrt{2} \rightarrow \sqrt{9-a}=2\sqrt{2}$

$9-a=8 \rightarrow \boxed{a=1}$

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$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=5 \rightarrow \frac{\sqrt{\alpha}+\sqrt{\beta}}{\sqrt{\alpha\beta}}=5 \rightarrow \sqrt{\alpha}+\sqrt{\beta}=5\sqrt{\alpha\beta} \qquad S+2\sqrt{P}=25P \rightarrow S+2\sqrt{\frac{1}{36}}=\underbrace{}_{\frac{25}{36}}$ ۴

$S=\frac{25}{36}-\frac{1}{3}=\frac{13}{36} \qquad \frac{m+14}{36}=\frac{13}{36} \rightarrow \boxed{m=-1} \rightarrow mx^2+3x+2$

$\rightarrow -x^2+3x+2 \rightarrow P=\frac{2}{-1}=\boxed{2}$