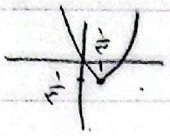
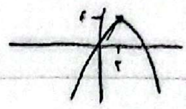
سارا عزیزی

۱۸/۷۵

① ۲

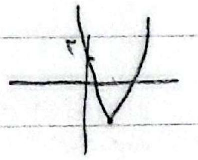
الف) $y=3x^2-2x \to a>0 , ext \left| \begin{matrix} \frac{2}{6}=\frac{1}{3} \\ -\frac{1}{3} \end{matrix} \right.$ ب) $y=-x^2+4x \to a<0 , ext \left| \begin{matrix} 2 \\ 4 \end{matrix} \right.$

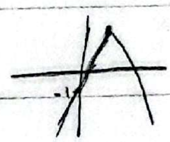
 → ناحیه ۳

 → ناحیه ۲

② ۳

الف) $y=2x^2-5x+2 \to a>0 , ext \left| \begin{matrix} \frac{5}{4} \\ \frac{-9}{8} \end{matrix} \right.$ ب) $-x^2+4x-1 \to a<0 , ext \left| \begin{matrix} 2 \\ 3 \end{matrix} \right.$

 → ۱و۲و۴

 → ۱و۳و۴

③ ۳

$x^2-x-3=0 \to S=1 \quad P=-3 \quad |\alpha-\beta| = \frac{\sqrt{1+12}}{1} = \sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{S}{|\alpha-\beta|} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$ ✓

ب) $\alpha^2+\beta^2 = S^2-2P = 1-(-6) = 7$ ✓

ج) $\alpha^3+\beta^3 = S^3-3SP = 1-3(-3\times 1) = 10$ ✓

د) $\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = \sqrt{13}(7-3)$

$= 4\sqrt{13}$

④ ۰/۷۵

$x=2$ می‌تواند ریشه‌ی این عبارت باشد

$y=(x-2)(x^2-ax+a)$ $\quad (x-2)^2 = x^2-4x+4 \quad a=4 \quad \mathrm{I}$

$x=2$ ← یا ← بدون ریشه ← $\Delta<0 \to a^2-4a<0 \to a(a-4)<0 \to \begin{matrix} 0 \quad 4 \\ + \; - \; + \end{matrix} \to a\in(0,4) \quad \mathrm{II}$

$\mathrm{I}\cup\mathrm{II} = (0,4]$

⑤ ۲

$3x^2-12x-a=0 \to S=\frac{12}{3}=4=\alpha+\beta \to \alpha=4-\beta$

$2\alpha^2+\beta^2-4\alpha=7 \to 2(4-\beta)^2+\beta^2-4(4-\beta) = 2(16+\beta^2-8\beta)+\beta^2-4(4-\beta) =$

$32+2\beta^2-16\beta+\beta^2-16+4\beta = 3\beta^2-12\beta+16 = 7 \to \frac{3\beta^2}{3}-\frac{12\beta}{3}+\frac{9}{3}=0$

$\to \beta^2-4\beta+3=0 \to (\beta-3)(\beta-1)=0 \to \beta=3 , \beta=1$

$\alpha=1 \quad \alpha=3$

$\to 3-12-a=0 \to \boxed{-9=a}$

۶ — ۳

$a-2 \in N \to a-2>0 \to a>2 \to 3,4,5,\dots$

$7-2a \in N \to 7-2a>0 \to 7>2a \to 3/5>a \to \dots,1,2,3$ $\Big\}\cap \to a=3$

$A(9,1) \quad B=(1,1) \quad b=\frac{9+1}{2}=\frac{10}{2}=5 \to S=(5,3)$

عرض برابر

$\left.\begin{aligned} &\to 1 = 81a+9b+c \\ &\ \ \ 1 = a+b+c \end{aligned}\right\} 80a+8b=0 \to 10a+b=0 \to b=-10a$

$\to 1 = 81a-90a+c \to 9a=c-1$

$3=25a+5(-10a)+c \to 3=25a-50a+c \to 25a=c-3$ $\Big\}$ $25a-9a=c-3-c+1 \to 16a=-2$

$\to a = -\frac{1}{8} \to -\frac{9}{8}=c-1 \to -\frac{9}{8}+\frac{8}{8}=c \to \frac{-1}{8}=c \to$ جواب $c=\frac{1}{8}$

۷ — ۳

$ax^2-ax-b=0 \to S=\frac{a}{a}=1,\ P=\frac{-b}{a}$

$\alpha+\beta=1 \to \alpha=\beta-1$

$40\beta^2+20(\beta-1)^2-20\beta=17 \to 40\beta^2+20\beta^2+20-40\beta-20\beta=17$

$60\beta^2-60\beta+3=0 \to 20\beta^2-20\beta-1=0 \to \beta^2-\beta+\frac{1}{20}=0 \to \Delta=1-\frac{1}{5}=\frac{4}{5}$

$|\alpha_1-\beta_2|=\frac{\sqrt{\Delta}}{|a|}=\frac{\frac{2}{\sqrt5}}{1}=\frac{2\sqrt5}{5}$

۸ — ۳

$-\frac{a+1}{2}=-2=x_{\text{رأس}} \to y_{\text{رأس}}=-\frac{1}{2},\ c=\frac{3}{2}$

$4a-2b+\frac{3}{2}=-\frac{1}{2} \to b-2b=-\frac{1}{2}-\frac{3}{2} \to b=2 \to a=\frac{2}{4}=\frac{1}{2}$

$25a-5b+\frac{3}{2}=a+b+\frac{3}{2} \to 24a=b \to \frac{1}{2}x^2+2x+\frac{3}{2}=y \to \frac{1}{2}+\frac{4}{2}+\frac{3}{2}=\boxed{4=\beta}$

۹ — ۳

$x^2+6x+a=0 \to \alpha=-3+\sqrt{9-a},\ b=-3-\sqrt{9-a}$

$\alpha^2=18-a-6\sqrt{9-a} \quad \beta^2=18-a+6\sqrt{9-a}$

$\to 3\alpha^2+2\beta^2=90-5a-6\sqrt{9-a}=12\sqrt2+85 \to 5a+6\sqrt{9-a}=5+6\sqrt8$

Scbd $\quad 9-a=8 \to \boxed{a=1}$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = \frac{\frac{\sqrt{m+۲۶}}{۴}}{\frac{1}{۴}} = \sqrt{m+۲۶} = ۵ \to m+۲۶ = ۲۵ \to m = -1$$ ①

$$(\sqrt{\alpha} + \sqrt{\beta})^۲ = \alpha + \beta + ۲\sqrt{\alpha\beta} = \frac{m+۱۶}{۳۶} + ۲\sqrt{\frac{1}{۳۶}} = \frac{m+۱۶}{۳۶} + \frac{1}{۳} = \frac{m+۲۸}{۳۶} \to \sqrt{\alpha} + \sqrt{\beta} = \frac{\sqrt{m+۲۸}}{۶}$$ ۴

$$\to mx^۲ + ۳x + ۲ = -x^۲ + ۳x + ۲ = 0 \to \beta = \frac{۲}{-1} = -۲$$