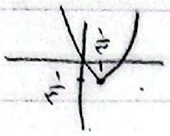
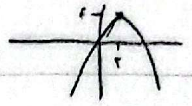
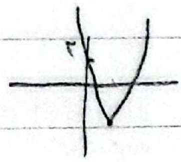
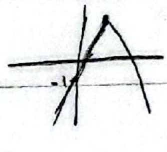
سارا عزیزی

① الف) $y=3x^2-2x \to a>0$, ext $\left|\begin{matrix}\frac{3}{9}=\frac{1}{3}\\ -\frac{1}{3}\end{matrix}\right.$ ب) $y=-x^2+4x \to a<0$, ext $\left|\begin{matrix}2\\ 4\end{matrix}\right.$

 → ناحیه ۳  → ناحیه ۲

② الف) $y=2x^2-5x+2 \to a>0$, ext $\left|\begin{matrix}\frac{5}{4}\\ \frac{-9}{8}\end{matrix}\right.$ ب) $-x^2+4x-1 \to a<0$, ext $\left|\begin{matrix}2\\ 3\end{matrix}\right.$

 → ۱و۲و۴  → ۱و۳و۴

③ $x^2-x-3=0 \to S=1 \quad P=-3 \quad |\alpha-\beta|=\sqrt{1+12}=\sqrt{13}$

الف) $\frac{\alpha+\beta}{\alpha-\beta}=\frac{S}{|\alpha-\beta|}=\frac{1}{\sqrt{13}}=\frac{\sqrt{13}}{13}$ ب) $\alpha^2+\beta^2=S^2-2P=1-(-6)=7$

ج) $\alpha^3+\beta^3=S^3-3SP=1-\cancel{3}\overset{-9}{(-3\times1)}=10$ د) $\alpha^3-\beta^3=(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta)=\sqrt{13}\,(7-3)$

$=4\sqrt{13}$

④ $y=(x-2)(x^2-ax+a)$

$x=2$ ↳ بدون ریشه $\to \Delta<0 \to a^2-4a<0 \to a(a-4)<0 \to \begin{matrix}0 & & 4\\ + & - & +\end{matrix} \to a\in(0,4)$

⑤ $3x^2-12x-a=0 \to S=\frac{12}{3}=4=\alpha+\beta \to \alpha=4-\beta$

$2\alpha^2+\beta^2-4\alpha=7 \to 2(4-\beta)^2+\beta^2-4(4-\beta)=2(16+\beta^2-8\beta)+\beta^2-4(4-\beta)=$

$32+2\beta^2-16\beta+\beta^2-16+4\beta=3\beta^2-12\beta+16=7 \to \frac{3\beta^2}{3}-\frac{12\beta}{3}+\frac{9}{3}=0$

$\to \beta^2-4\beta+3=0 \to (\beta-3)(\beta-1)=0 \to \beta=3,\ \beta=1$

$\alpha=1 \qquad \alpha=3$

↳ $3-12-a=0 \to \boxed{-9=a}$

(6)

$a-2\in N \to a-2>0 \to a>2 \to 3,4,5,\dots$

$7-2a\in N \to 7-2a>0 \to 7>2a \to 3/5>a \to \dots,1,2,3$ $\Big\}\cap \to a=3$

$\underbrace{A(9,1)\quad B=(1,1)}_{\text{عرض برابر}} \to b=\frac{9+1}{2}=\frac{10}{2}=5 \to S=(5,3)$

$\to 1=81a+9b+c$

$\phantom{\to} 1=a+b+c$ $\Big\}\ 80a+8b=0 \to 10a+b=0 \to b=-10a$

$\to 1=81a-90a+c \to 9a=c-1$

$3=25a+5(-10a)+c \to 3=25a-50a+c \to 25a=c-3$ $\Big\}\ 25a-9a=c-3-c+1 \to 16a=-2$

$\to a=-\frac{1}{8} \to -\frac{9}{8}=c-1 \to -\frac{9}{8}+\frac{8}{8}=c \to \frac{-1}{8}=c \to$ [illegible] $c=\frac{1}{8}$

(7)

$ax^2-ax-b=0 \to S=\frac{a}{a}=1,\ P=\frac{-b}{a}$

$\alpha+\beta=1 \to \alpha=\beta-1$

$40\beta^2+20(\beta-1)^2-20\beta=17 \to 40\beta^2+20\beta^2+20-40\beta-20\beta=17$

$60\beta^2-60\beta+3=0 \to 20\beta^2-20\beta-1=0 \to \beta^2-\beta+\frac{1}{20}=0 \to \Delta=1-\frac{1}{5}=\frac{4}{5}$

$|\alpha-\beta|=\frac{\sqrt{\Delta}}{|a|}=\frac{\frac{2}{\sqrt5}}{1}=\frac{2\sqrt5}{5}$

(8)

$-\frac{a+1}{2}=-2=x_{\text{رأس}} \to y_{\text{رأس}}=-\frac{1}{2},\ c=\frac{3}{2}$

$4a-2b+\frac{3}{2}=-\frac{1}{2} \to b-2b=-\frac{1}{2}-\frac{3}{2} \to b=2 \to a=\frac{2}{4}=\frac{1}{2}$

$25a-5b+\frac{3}{2}=a+b+\frac{3}{2} \to 4a=b \to \frac{1}{2}x^2+2x+\frac{3}{2}=y \to \frac{1}{2}+\frac{4}{2}+\frac{3}{2}=\boxed{4=\beta}$

(9)

$x^2+6x+a=0 \to \alpha=-3+\sqrt{9-a},\ b=-3-\sqrt{9-a}$

$\alpha^2=18-a-6\sqrt{9-a}\qquad \beta^2=18-a+6\sqrt{9-a}$

$\to 3\alpha^2+2\beta^2=90-5a-6\sqrt{9-a}=12\sqrt{2}+85 \to 5a+6\sqrt{9-a}=9+6\sqrt{8}$

$9-a=8 \to \boxed{a=1}$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}} = \frac{\frac{\sqrt{m+۲۶}}{۶}}{\frac{1}{۶}} = \sqrt{m+۲۶} = ۵ \to m+۲۶ = ۲۵ \to m = -1$$ ب

$$(\sqrt{\alpha} + \sqrt{\beta})^۲ = \alpha + \beta + ۲\sqrt{\alpha\beta} = \frac{m+۱۴}{۳۶} + ۲\sqrt{\frac{1}{۳۶}} = \frac{m+۱۴}{۳۶} + \frac{1}{۳} = \frac{m+۲۶}{۳۶} \to \sqrt{\alpha}+\sqrt{\beta} = \frac{\sqrt{m+۲۶}}{۶}$$

$$\to mx^۲ - ۳x + ۲ = -x^۲ + ۳x + ۲ = 0 \to P = \frac{۲}{-1} = -۲$$