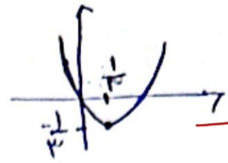
دهلیا عرب

۲۰

۱) الف) $y = 3x^2 - 2x$ $\quad \left| \begin{array}{l} \frac{-b}{2a} = \frac{2}{6} = \frac{1}{3} \\ y = \frac{1}{3} - \frac{2}{3} = -\frac{1}{3} \end{array} \right.$

عرض از مبدأ ۰



از ناحیه‌ی ۳ نمی‌گذرد

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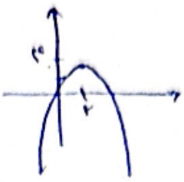
ب) $y = -x^2 + 4x$ $\quad \left| \begin{array}{l} \frac{-b}{2a} = 2 \\ y = 4 \end{array} \right.$



از ۳ ناحیه نمی‌گذرد

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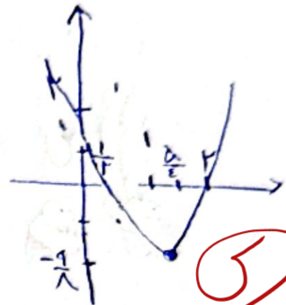
۲) الف) $y = 2x^2 - 5x + 2$ $\quad x = \frac{-b}{2a} = \frac{5}{4} \quad y = \frac{-\Delta}{4a} = \frac{-9}{8}$

ریشه‌ها $= \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{5 \pm \sqrt{9}}{4}$ ↗ $\frac{8}{4} = 2$ ↘ $\frac{2}{4} = \frac{1}{2}$

از ناحیه‌ی ۱ و ۲ و ۴



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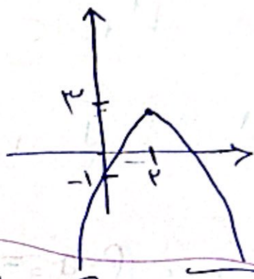
ب) $y = -x^2 + 4x - 1$ $\quad \left| \begin{array}{l} x = \frac{-b}{2a} = 2 \\ y = 3 \end{array} \right.$



از ۱ و ۳ و ۴

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۳) الف) $\frac{\alpha+\beta}{\alpha-\beta} \rightarrow S = \frac{-b}{a} = 1$

$\alpha - \beta$ یا $\frac{\sqrt{\Delta}}{|a|} = \sqrt{1-4(-3)(1)} = \sqrt{13}$ $\quad = \frac{1}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{\sqrt{13}}$

ب) $\alpha^2 + \beta^2 = S^2 - 2P \rightarrow 1 - 2(-3) = 7$

۲

ج) $\alpha^3 + \beta^3 = S^3 - 3P.S = 1 - 3(-3)(1) = 10$

د) $\alpha^3 - \beta^3 = (\alpha-\beta)^3 + 3\alpha.\beta(\alpha-\beta) = (\sqrt{13})^3 + 3(-3)(\sqrt{13}) = 13\sqrt{13} - 9\sqrt{13} = 4\sqrt{13}$

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۴) $y = (x-2)(x^2 - ax + a) \rightarrow x - 2 = 0$ ریشه‌ی

$x - 2 = 0$, $x = 2$

یا $\Delta < 0 \Rightarrow a^2 - 4a < 0 \rightarrow a(a-4) < 0$ $\quad 0 < a < 4$

$x^2 - ax + a \overset{x=2}{\Rightarrow} 4 - 2a + a = 0 \rightarrow -a = -4 \rightarrow a = 4$ $\quad 0 < a \leq 4$

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$3x^2 - 12x + 9 - a = 0$ $\quad 3x^2 - 12x + 9 \Rightarrow$ ① a, ③ $\frac{c}{a}$ $\quad \frac{-9}{3} = -3$ برابر

$2\alpha^2 + \beta^2 - 4\alpha = 7$

$\beta^2 + \alpha^2 + \alpha^2 - 4\alpha = 7$

↓ $4\alpha + \frac{a}{3}$ $\quad \alpha^2 + \beta^2 = 16 + \frac{2a}{3} \Rightarrow S^2 - 2PS =$

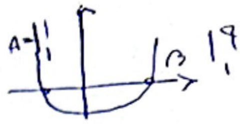
$16 + \frac{2a}{3} + \ldots = 7 \rightarrow \frac{2a}{3} = -6$

$a = -9$

۲

۶) $A = (2a+3 , a-2) , B(7-2a , a-2)$ برابر برابر $\quad y = a(x-5)^2 + 3$

x رأس $= \frac{2a+3+7-2a}{2} = 5$ $\quad \Big|^{5}_{3}$ $\quad 1 = 16a+3$



$b = 5 \quad b - 2 = 3$

$a = -\frac{1}{8}$

$y = 0 = -\frac{1}{8}(x-5)^2 + 3$

$y(0) = -\frac{25}{8} + 3 = -\frac{1}{8} \Rightarrow$ فاصله $\frac{1}{8}$ فاصله مبدأ

③

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۷) $ax^2 - ax - b = 0$ $\quad \alpha + \beta = 1$

$40\beta^2 + 20\alpha^2 - 20\beta = 17$ $\quad \beta^2 = (1-\alpha)^2$

$40(1-\alpha)^2 + 20(\alpha^2) - 20(1-\alpha) = 17 \rightarrow 60\alpha^2 - 60\alpha + 3 = 0$

$S = 1$, $P = \frac{1}{20}$

$|\alpha - \beta| = \sqrt{S^2 - 4P} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

②

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۸) $(-5, \beta) , (b, \beta) \rightarrow \frac{-5+(1)}{2} = -2 \rightarrow x = -2$ $\quad ext \Big|^{-2}_{-\frac{1}{2}}$ $\quad \rightarrow -\frac{b}{2a} = -2$

$-\frac{\Delta}{4a} \Rightarrow \frac{-b^2+4ac}{b} = \frac{-b+c}{\frac{3}{2}} = -\frac{1}{2} \Rightarrow b = 2$ $\quad b = 4a$

③

$\frac{1}{2}x^2 + 2x + \frac{3}{2} \xrightarrow{x=1} \frac{1}{2} + 2 + \frac{3}{2} \Rightarrow 4 = b$

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۹) $x^2 + 6x + a = 0$ $\quad \begin{cases} \alpha = -3 + \sqrt{9-a} \rightarrow \alpha^2 = 18 - a - 6\sqrt{9-a} \\ \beta = -3 - \sqrt{9-a} \rightarrow \beta^2 = 18 - a + 6\sqrt{9-a} \end{cases}$

$3\beta^2 + 2\beta^2 = 90 - 5a - 6\sqrt{9-a} = 12\sqrt{2} + 85$

$5a + 6\sqrt{9-a} = 5 + 6\sqrt{8} \Rightarrow a = 1$

④

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۱۰) $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = 5 \Rightarrow \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = 5$

③

$\alpha \cdot \beta = \frac{1}{36} , \alpha + \beta = \frac{m+12}{36}$ $\quad (\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha \cdot \beta}$

$\frac{m+12}{36} + \frac{2}{6} = \frac{m+24}{36}$

$6 \times \frac{\sqrt{m+24}}{6} = 5$

$m + 24 = 25 \Rightarrow m = -1$ $\quad \frac{\sqrt{m+24}}{6} = \sqrt{\alpha} + \sqrt{\beta}$

$\alpha_2 \cdot \beta_2 \longrightarrow mx^2 + 3x + 2 \rightarrow \frac{c}{a} = \frac{2}{m} = -2$