۱۳/۵

۱) **شکل سهمی ها باید کشیده شود** **۱/۵**

الف) $y = 3x^2 - 2x$ — $a>0 \to ext: min$

$x(3x-2)=0$ — $x<0 \to y>0$ ②

$x=0 \quad x=\frac{2}{3}$ — $0<x<\frac{2}{3} \to y<0$ ④

$x>\frac{2}{3} \to y>0$ ①

از ناحیه [illegible] ② عبور نمی کند

ب) $y = -x^2 + 4x$

$-x(x-4)=0$ — $a<0 \to ext: max$

$x=0 \quad x=4$

$x<0 \to y<0$ ③

$0<x<4 \to y>0$ ①

$x>4 \to y<0$ ④

از ناحیه ② عبور نمی کند.

۲) **شکل سهمی باید کشیده شود** **۱/۵**

الف) $2x^2-5x+2$

$\Delta = 25-16=9$ — $a>0 \to ext: min$

$x=\frac{5\pm 3}{4} = 2, \frac{1}{2}$

$x<0 \to y>0$ ②

$\frac{1}{2}<x<2 \to y<0$ ④

$x>2 \to y>0$ ①

از نواحی اول، دوم و چهارم می گذرد.

ب) $y=-x^2+4x-1$ — $a<0 \to ext: max$

$\Delta = 16-4=12$

$x<0 \to y<0$ ③

$x=\frac{4\pm 2\sqrt{3}}{2} = 2\pm\sqrt{3}$ — $2-\sqrt{3}<x<2+\sqrt{3} \to y>0$ ①

$x>2+\sqrt{3} \to y<0$ ④

از نواحی اول، سوم و چهارم می گذرد.

۳) **۵/۵**

الف) $\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\frac{\sqrt{\Delta}}{|a|}} = \frac{|a|}{\sqrt{\Delta}} = \frac{1}{\sqrt{1+12}} = \frac{\sqrt{13}}{13}$ ✓

ب) $\alpha^2+\beta^2 =$ ~~$(\alpha+\beta)^2-2\alpha\beta = 1-2(3) = 1-6=-5$~~ $S^2-2P = (1)^2-2(-3) = 7$

ج) $\alpha^3+\beta^3 =$ ~~$(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta) = 1-3(-3)(1) = 1-9=-8$~~ $S^3-3PS = 1^3-3(1)(-3) = 10$

د) $\alpha^3-\beta^3 = (\alpha-\beta)^3+3\alpha\beta(\alpha-\beta) =$ ~~$(\sqrt{13})^3+3(-3)(\sqrt{13}) = 13\sqrt{13}+9\sqrt{13} = 22\sqrt{13}$~~

$(\alpha-\beta)(\alpha^2+\beta^2+\alpha\beta) = \sqrt{13}(7-3) = 4\sqrt{13}$

۴) **۲**

$(x-2)(x^2-ax+a)=0 \to y=0$

$2 \quad x^2-ax+a=0$ — $x=2$

$\Delta<0 \quad a^2-4a<0$ — $2^2-2a+a=0 \quad 4-a=0, \ a=4$

$a(a-4)<0 \to 0<a<4$ $\} \to 0<a\leq 4$

۵) **۳**

$3x^2-12x-a=0$ — $\beta = 4-\alpha$

$\alpha+\beta = -\frac{b}{a} = \frac{12}{3} = 4$ — $\alpha\beta = \frac{c}{a} = -\frac{a}{3}$

$2\alpha^2+\beta^2-4\alpha=7$

$2\alpha^2+(4-\alpha)^2-4\alpha=7$ — $2\alpha^2+16+\alpha^2-8\alpha-4\alpha=7$ — $3\alpha^2-12\alpha+9=0$

$\alpha^2-4\alpha+3=0$ — $(\alpha-1)(\alpha-3)=0$

$\alpha=1 \quad \alpha=3$

$\beta=3 \quad \beta=1$

$\alpha\beta = 3\times 1 = 3 = \frac{-a}{3} \to -a=9 \quad a=-9$

$\frac{-9}{3} = -3$

۶) **۳**

$x=b$

$|2a+3-b| = |7-2a-b|$

~~$2a$~~ $+3-b = -7+$ ~~$2a$~~ $+b$

$2b=10 \quad b=5$ — $S=(5, b-2) \to S=(5,3)$

$y=k(x-5)^2+3$

$a-2 = k(2a+3-5)^2+3$

$a-2 = k(2a-2)^2+3$

$a-2-3 = k(2a-2)^2$

$3-5 = k(4)^2$

$16k=-2$

$k=-\frac{1}{8}$

$y=-\frac{1}{8}(x-5)^2+3$

$x=0$

$y=-\frac{1}{8}(0-5)^2+3$

$y=-\frac{25}{8}+\frac{24}{8} = -\frac{1}{8}$ → [illegible] : $(0,-\frac{1}{8})$

$d=\sqrt{0+(-\frac{1}{8})^2} = \frac{1}{8}$

$\alpha+\beta=\frac{a}{a}=1 \quad \alpha\beta=\frac{b}{a}$

$\hookrightarrow \beta=1-\alpha$

$ax^2-ax-b=0$

$|\alpha-\beta|=\frac{\sqrt{a^2+4ab}}{a}=\sqrt{1+\frac{4b}{a}}$

$|\alpha-\beta|=\frac{\sqrt{b^2-4ac}}{a}$

(۷)

$40\beta^2+20\alpha^2-20\beta=17$

$40(1-\alpha)^2+20\alpha^2-20(1-\alpha)=17$

$40(\alpha^2-2\alpha+1)+20\alpha^2-20+20\alpha=17$

$60\alpha^2-40\alpha+20=17 \longrightarrow 60\alpha^2-40\alpha+3=0 \longrightarrow 20\alpha^2-20\alpha+1=0$

$\Delta=400-80=320$

$|\alpha_1-\alpha_2|=\frac{\sqrt{\Delta}}{|a|}=\frac{8\sqrt{a}}{20}=\frac{2\sqrt{a}}{5}$

(۸)

$y=ax^2+bx+c$

$-\frac{b}{2a}=-\frac{1}{2} \longrightarrow a=b \qquad x=0 \longrightarrow y=c=\frac{3}{2}$

$x=1 \longrightarrow \beta=a+a+c=2a+c=2a+\frac{3}{2}$

$x=-2 \longrightarrow \beta=4a-2a+c=2a+c=2a+\frac{3}{2}$

$2a+\frac{3}{2}=20a=\frac{5}{2}$

$4a=0 \quad a=0 \times$

$\hookrightarrow \beta=\frac{3}{2}$

$x=-2\pm\sqrt{9-a}$

$\alpha=-2-\sqrt{9-a} \qquad \beta=-2+\sqrt{9-a}$

$\alpha^2=18-a+4\sqrt{9-a}$

$\beta^2=18-a-4\sqrt{9-a}$

$90-8a+4\sqrt{9-a}=8\alpha+12\sqrt{2}$

$90-20a=80$

$a=1$

(۹)

$\frac{1}{x_1}+\frac{1}{x_2}=2 \qquad \frac{x_2+x_1}{x_2x_1}=2 \longrightarrow \frac{\frac{m-14}{24}}{\frac{1}{24}}=\frac{m-14}{24}\times\left(\frac{24}{1}\right)=m-14=2$

$m=19$

$19x^2+2x+2=0 \longrightarrow x_1x_2=\frac{c}{a}=\frac{2}{19}$

(۱۰)

سوال ۱۸

$x_S=\frac{-a+1}{2}=-2$

$f(x)=a(x+2)^2-\frac{1}{2}$

$(0,\frac{3}{2})\in f(x) \longrightarrow \frac{3}{2}=a(0+2)^2-\frac{1}{2} \longrightarrow a=\frac{1}{2}$

$(1,\beta)\in f(x) \longrightarrow \beta=\frac{1}{2}(1+2)^2-\frac{1}{2} \longrightarrow \beta=4$

$A=\sqrt{\frac{1}{x_1}}+\sqrt{\frac{1}{x_2}}=3 \longrightarrow A^2=\frac{1}{x_1}+\frac{1}{x_2}+2\sqrt{\frac{1}{x_1x_2}}$

(۱۵)

$\frac{x_1+x_2}{x_1x_2}+2\sqrt{\frac{1}{x_1x_2}}=\frac{5}{6}+2\sqrt{\frac{1}{6}}=2a$

$m+14+2(4)=24 \longrightarrow m=-1$

$\text{حد}=\frac{c}{a}=\frac{2}{m}=\frac{2}{-1}=-2$