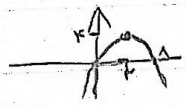
۱۸/۱۰/۹۳

آنا شکرچی

۱- الف) $y = 3x^2 - 2x$ $x = \frac{-b}{2a} = \frac{2}{6}$ ، $y = \frac{-\Delta}{4a} = \frac{-4}{12}$ عرض از مبدأ صفر

ب) $y = -x^2 + 4x$ $x = \frac{-b}{2a} = 2$ ، $y = 4$ max دارد ← سهمی ریشه‌ها صفر و دهانه ↓

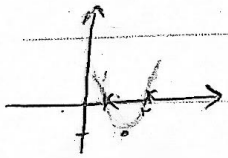
ناصبه دوم



ناصبه سوم

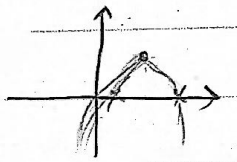
۲- الف) $y = 2x^2 - 5x + 2$ $x = \frac{-b}{2a} = \frac{5}{4}$ ، $y = \frac{-\Delta}{4a} = \frac{-9}{8}$

ریشه‌ها $= \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{5 \pm \sqrt{9}}{4}$ $\begin{cases} \frac{8}{4} = 2 \\ \frac{2}{4} = \frac{1}{2} \end{cases}$ → ناصبه اول و ۲



ب) $y = -x^2 + 4x - 1$ $x = \frac{-b}{2a} = \frac{-4}{-2} = 2$ ، $y = \frac{-\Delta}{4a} = \frac{-b^2 + 4ac}{-4} = \frac{-12}{-4} = 3$

ریشه‌ها $= \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{-4 \pm \sqrt{12}}{-2} = \begin{cases} \frac{-4+2\sqrt{3}}{-2} = 2 - \sqrt{3} \\ \frac{-4-2\sqrt{3}}{-2} = 2 + \sqrt{3} \end{cases}$



→ ناصبه اول و ۳ و ۴

۳- الف) $\frac{\alpha+\beta}{\alpha-\beta} = (x^2 - x - 3 = 0)$ $\alpha+\beta = \frac{-b}{a} = 1$ ، $\alpha\beta = \frac{c}{a} = -3$

$\alpha-\beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{1+12}}{1} = \sqrt{13}$

$\frac{1}{\sqrt{13}} \times \frac{\sqrt{13}}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

ب) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 1 - 2 \times -3 = 7$

ج) $= \alpha^3 + \beta^3 = S^3 - 3SP = 1^3 - 3 \times 1 \times -3 = 10$

د) $= \alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \beta^2 + \alpha\beta) = \sqrt{13}(7 - 3) = 4\sqrt{13}$

($\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$ ، $\alpha^2 + \beta^2 = 7$ ، $\alpha\beta = -3$)

۴) ۸/۱۷۵

$y=(x-2)(x^2-4x+a)$ → یکی از ریشه‌ها ۲

ریشه مضاعف ندارد

$I \cup II = (0, 4]$

$\Delta < 0 \quad a^2-4a<0 \quad a(a-4)<0$

مجموعه مقادیر a → $(0,4) \pm$

	0		4	
	+	−	0	+

$x=2 \to (x-2)^2 = x^2-4x+4 \to a=4$ II

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۵) ۲

$3x^2-11x-a=0 \quad , \quad 2\alpha^2+\beta^2-4\alpha=7$

$\beta^2+\alpha^2 = S^2-2P = 11^2-2(\frac{-a}{3}) = 14+\frac{2a}{3} = \frac{48+2a}{3}$

$\alpha^2 = \frac{12\alpha+a}{3}$

$\hookrightarrow \alpha^2+\alpha^2+\beta^2-4\alpha-7=0 \to \frac{12\alpha+a}{3}+\frac{48+2a}{3}-4\alpha-7=0$

$\hookrightarrow 4\alpha+14+a-4\alpha-7 \Rightarrow a = 7-14 = -9$

ریشه‌ها $\to \frac{-b\pm\sqrt{\Delta}}{2a} = \frac{11\pm\sqrt{121-108}}{6} = \begin{cases} \frac{11+6}{6}=3 \to \text{بزرگ‌تر} \\ \frac{11-6}{6}=1 \end{cases}$ نسبت $= \frac{a}{3} = \frac{-9}{3} = \boxed{-3}$

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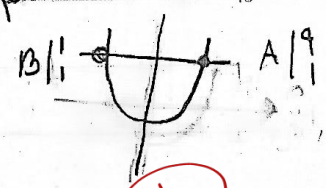
۶) ۴ ـ عددهای برابر دارند

$A(2a+3, a-2) \quad , \quad B(7-2a, a-2)$

وسط $S\left(\frac{b}{2}, \frac{b+2}{3}\right)$ $\quad ext\left|x=\frac{2a+3+7-2a}{2}=5\right.$ محور تقارن

$S \mid \begin{matrix} 5 \\ 3 \end{matrix}$



۲

$y=a(x-5)^2+3$

$1=\frac{14a+3}{2} \Rightarrow \boxed{a=\frac{-1}{8}}$

$y(0) = \frac{-1}{8}(x-5)^2+3$

$y(0) = \frac{-25}{8}+3 = \frac{-1}{8} \to$ چون فاصله‌ی خواسته شده مثبت است $\left(\frac{1}{8}\right)$

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۷) ۳

$9x^2-9x-b=0 \quad , \quad 45\beta^2+25\alpha^2-25\beta=17$

$\hookrightarrow \beta+\alpha=1, \quad \beta\alpha=\frac{-b}{9}$ / $45\beta^2+25\overbrace{(1-\beta)^2}^{1-2\beta+\beta^2}-25\beta=17$

$|\alpha-\beta| = 1-2\beta = 1-2\times\left(\frac{5\pm2\sqrt{5}}{10}\right)$

$45\beta^2+25-45\beta+25\beta^2-25\beta=17$

$70\beta^2-70\beta+8=0 \to 25\beta^2-25\beta+1=0$

$\hookrightarrow \left|\frac{\pm2\sqrt{5}}{5}\right| = \boxed{\frac{2\sqrt{5}}{5}}$

$\beta = \frac{25\pm\sqrt{625-100}}{50} \Rightarrow \frac{25\pm8\sqrt{5}}{50} \to \frac{5\pm2\sqrt{5}}{10}$

۸- $\frac{-\Delta+1}{2} = \frac{-b}{2a} = -2 \longrightarrow 2a = b$

$\frac{-\Delta}{4a} = \frac{-b^2 + 4ac}{b} = -b + \frac{4c}{b} = \frac{-1}{2} \longrightarrow -b = \frac{-1}{2} - \frac{4}{2} = -2 \longrightarrow b = 2$

$\frac{1}{2}x^2 + 2x + \frac{3}{2} \xrightarrow{x=1} \frac{1}{2} + 2 + \frac{3}{2} = \sqrt{4} = \beta$

۹- $x^2 + 4x + a = 0 \qquad a < \beta < 0 \qquad \alpha + \beta = -4$

$\alpha\beta = a$

$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 18$

$x^2 + 4x + a = 0 \begin{cases} \alpha = -2 + \sqrt{9-a} \rightarrow \alpha^2 = 18 - a - 4\sqrt{9-a} \\ \beta = -2 - \sqrt{9-a} \rightarrow \beta^2 = 18 - a + 4\sqrt{9-a} \end{cases}$

$3\alpha^2 + 2\beta^2 = 90 - 5a - 4\sqrt{9-a} = 12\sqrt{2} + 18 \longrightarrow 5a + 4\sqrt{9-a} = 5 + 4\sqrt{18}$

$\hookrightarrow \boxed{a = 1}$

۱۰- $\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \left(\frac{\sqrt{\beta} + \sqrt{\alpha}}{\sqrt{\alpha\beta}}\right)$

$\alpha\beta = \frac{1}{3y}$

$\alpha + \beta = \frac{m+12}{3y}$

$\frac{\sqrt{\beta} + \sqrt{\alpha}}{\frac{1}{y}} = (y \times \sqrt{\alpha} + \sqrt{\beta} =$

$(\sqrt{\alpha} + \sqrt{\beta})^2 = \alpha + \beta + 2\sqrt{\alpha\beta} =$

$\frac{m+12}{3y} + \frac{2}{y \times} = \frac{m+2y}{3y}$

$\hookrightarrow y \times \frac{\sqrt{m+2y}}{y} = 5$

$\hookrightarrow \frac{\sqrt{m+2y}}{y} = \sqrt{\alpha} + \sqrt{\beta}$

$m + 2y = 25 \longrightarrow \boxed{m = -1}$

$\alpha_1 \beta_1 \xrightarrow[\text{معادله}]{} mx^2 + 3x + 2 = 0$ حاصل ضرب ریشه‌ها: $\frac{c}{a} = \frac{2}{m} = \frac{2}{-1} = \boxed{-2}$