Page: (   )

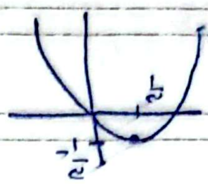
[illegible]

SUBJECT

Year: Month: Day:

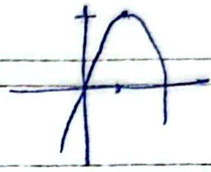
1)

الف) $y = 3x^2 - 2x$ $\quad a > 0$ و $\left| \begin{matrix} \frac{1}{3} \\ -\frac{1}{3} \end{matrix} \right.$ $\quad c = 0$



$\Rightarrow$ از ناحیه ۳ نمی‌گذرد

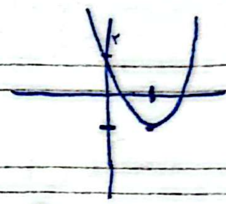
ب) $y = -x^2 + 4x$ $\quad a < 0$ و $\left| \begin{matrix} 2 \\ 4 \end{matrix} \right.$ $\quad c = 0$



$\Rightarrow$ از ناحیه ۳ نمی‌گذرد

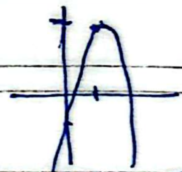
2)

الف) $y = 2x^2 - 5x + 2$ $\quad a > 0$ $\left| \begin{matrix} \frac{5}{4} = 1\frac{1}{4} \\ \frac{-9}{8} = -1\frac{1}{8} \end{matrix} \right.$ $\quad c = 2$



① و ② و ④

ب) $y = -x^2 + 4x - 1$ $\quad a < 0$ $\left| \begin{matrix} 2 \\ 3 \end{matrix} \right.$ $\quad c = -1$



① و ③ و ④

3) $\alpha + \beta = 1$ ، $\alpha\beta = -3$ و $|\alpha - \beta| = \frac{\sqrt{\Delta}}{|a|} = \sqrt{13}$

1- $\frac{\alpha + \beta}{\alpha - \beta} = \frac{\sqrt{13}}{13}$

2- $\alpha^2 + \beta^2 = S^2 - 2P = 7$

3- $\alpha^3 + \beta^3 = S^3 - 3PS = 10$

4- $\alpha^3 - \beta^3 = \underbrace{(\alpha - \beta)^3}_{13\sqrt{13}} + \underbrace{3\alpha\beta}_{-9}\underbrace{(\alpha - \beta)}_{\sqrt{13}} = 4\sqrt{13}$

④

$y = x^3 - x^2(a+2) + x(3a) - 2a$

$\Delta < 0 \quad a^2 - 4a < 0 \quad a(a-4) < 0 \Rightarrow 0 < a < 4$ $\quad 0 < a \leqslant 4$

$(x-2)^2 = x^2 - 4x + 4 = x^2 - ax + a \quad a = 4$

⑤

$2\alpha^2 + \beta^2 - 4\alpha = 7 \quad , \quad 3x^2 - 12x - a = 0$

$\alpha + \beta = 4 \quad , \quad \alpha\beta = \frac{-a}{3}$

$\beta = 4 - \alpha$

$2\alpha^2 + (4-\alpha)^2 - 4\alpha = 7 \Rightarrow 3\alpha^2 - 12\alpha + 9 = 0 \Rightarrow \alpha^2 - 4\alpha + 3 = 0 \Rightarrow \alpha = 1 , 3$

$\alpha\beta = \frac{-a}{3} = 3 \Rightarrow a = -9 \Rightarrow \frac{a}{\text{...}} = \frac{-9}{3} = -3$

⑥

$y_A = y_B = a - 2 \rightarrow$ متقارن $\Rightarrow b = \frac{2a + 3 + 7 - 2a}{2} = \frac{10}{2} = 5 \Rightarrow \boxed{b = 5}$

$\Rightarrow S(5, 3) \qquad a - 2 \in \mathbb{N} \Rightarrow a - 2 \geqslant 1 \Rightarrow a \geqslant 3$

$7 - 2a \geqslant 1 \Rightarrow 2a \leqslant 6 \Rightarrow a \leqslant 3 \Rightarrow a = 3$

$2a + 3 \neq 7 - 2a \Rightarrow \boxed{a \neq 1} \qquad \Rightarrow A(9, 1) , B(1, 1) , S(5, 3)$

$y = k(x-5)^2 + 3 \xrightarrow{A(9,1)} 1 = k(9-5)^2 + 3 = 16k + 3 \Rightarrow k = -\frac{1}{8}$

$y = -\frac{1}{8}(x-5)^2 + 3 \xrightarrow{x=0} -\frac{1}{8} \Rightarrow (0, -\frac{1}{8})$

$d = \sqrt{(0-0)^2 + (-\frac{1}{8} - 0)^2} = \sqrt{(-\frac{1}{8})^2} = \frac{1}{8}$

کوئیزهای بچه‌ها

SUBJECT

Year: Month: Day:

(۷)

$\alpha+\beta=1 \quad , \quad \alpha\beta=\frac{-b}{a}$

$\alpha = 1-\beta$

$40\beta^2+20(1-\beta)^2-20\beta=17 \Rightarrow 20\beta^2-20\beta+1=0$

$(\alpha-\beta)^2=(\alpha+\beta)^2-4\alpha\beta$

$\alpha\beta=(1-\beta)\beta=\beta-\beta^2 \Rightarrow \beta^2-\beta+\frac{1}{20}=0$

$\Rightarrow \alpha\beta=\frac{1}{20}$

$\Rightarrow (\alpha-\beta)^2=1-\underbrace{4\left(\tfrac{1}{20}\right)}_{=\frac{1}{5}} \Rightarrow |\alpha-\beta|=\sqrt{\frac{4}{5}} \Rightarrow \frac{2}{\sqrt{5}}=\frac{2\sqrt{5}}{5}$

(۸)

$\frac{-5+1}{2}=-2=\frac{-b}{2a} \Rightarrow b=4a$

$y=ax^2+bx+\frac{5}{2} \longrightarrow -\frac{1}{2}-\frac{5}{2}=4a+2b \Rightarrow -2=b-2b$

$\Rightarrow b=2 \Rightarrow a=\frac{1}{2}$

$y=\frac{1}{2}x^2+2x+\frac{5}{2} \xrightarrow{x=1} \frac{1}{2}+2+\frac{3}{2}=4=\beta$

(۹)

$x^2+6x+a=0 \qquad \alpha<\beta<0 \qquad 3\alpha^2+2\beta^2=12\sqrt{2}$

$\alpha,\beta=\frac{-b\pm\sqrt{\Delta}}{2a}=-3\pm\sqrt{9-a}$

$\Rightarrow 3(-3-\sqrt{9-a})^2+2(-3+\sqrt{9-a})^2=3(9+9-a+6\sqrt{9-a})+2(9+9-a-6\sqrt{9-a})$

$90-5a+6\sqrt{9-a}=12\sqrt{2}+85$

$5-5a+6\sqrt{9-a}=12\sqrt{2} \xrightarrow{a=1} 6\sqrt{8}=6\sqrt{4\times2}=12\sqrt{2}$ ✓

$\Rightarrow a=1$

$\alpha,\beta=\frac{-6\pm\sqrt{36-40}}{2}=-3\pm\sqrt{9-4}$

$5a=5 \Rightarrow a=1$

(۱۰)

$36x^2-(m+14)x+1=0$

$\alpha+\beta=\frac{-b}{a}=\frac{m+14}{36}$ $\alpha\beta=\frac{c}{a}=\frac{1}{36}$ = ریشه‌ها هم علامتند

$\frac{1}{\sqrt{\alpha}}+\frac{1}{\sqrt{\beta}}=5 \Rightarrow \frac{\sqrt{\beta}+\sqrt{\alpha}}{\sqrt{\alpha\beta}}=5$

$\Rightarrow \frac{\sqrt{\alpha}+\sqrt{\beta}}{\frac{1}{6}}=5 \Rightarrow \sqrt{\alpha}+\sqrt{\beta}=\frac{5}{6}$ $\xrightarrow{\text{توان ۲}}$ $\alpha+\beta+2\sqrt{\alpha\beta}=\frac{25}{36}$

$\Rightarrow \left(\frac{m+14}{36}\right)+\frac{1}{3}=\frac{25}{36} \Rightarrow m+14+12=25 \Rightarrow \boxed{m=-1} \Rightarrow \frac{12}{36}$

$mx^2+3x+2=0 \longrightarrow -x^2+3x+2=0$ یا $x^2-3x-2=0$

$\alpha_2\beta_2=\frac{c}{a}=\boxed{-2}$