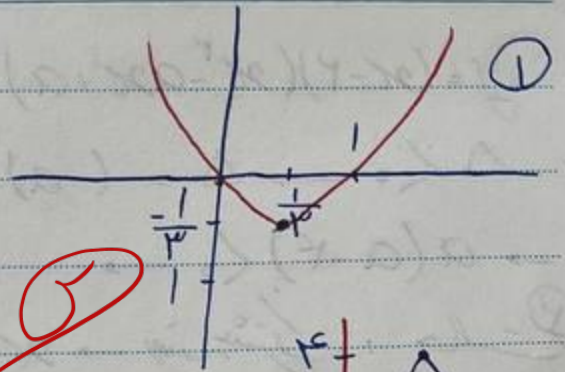


الف) $y = 3x^2 - 2x \rightarrow \min \left| \frac{-b}{2a} = \frac{1}{3} \right.$

$$\Delta = 4 - (4 \times 3 \times 0) = 4$$

از ناحیه ۳ نمی‌گذرد.

$$\left| \frac{-\Delta}{4a} = \frac{-4}{12} = -\frac{1}{3} \right.$$

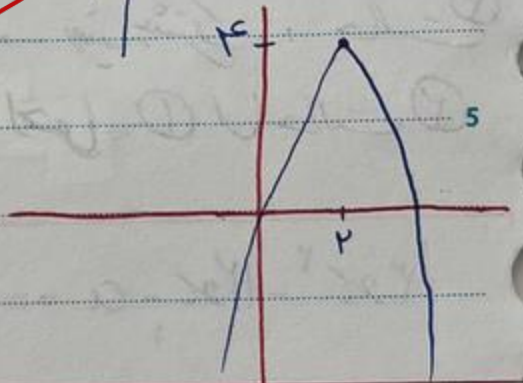


ب) $y = -x^2 + 4x \rightarrow \max \left| \frac{-b}{-2a} = 2 \right.$

$$\Delta = 16 - 0 = 16$$

از ناحیه ۲ نمی‌گذرد.

$$\left| \frac{-\Delta}{4a} = \frac{-16}{-4} = 4 \right.$$

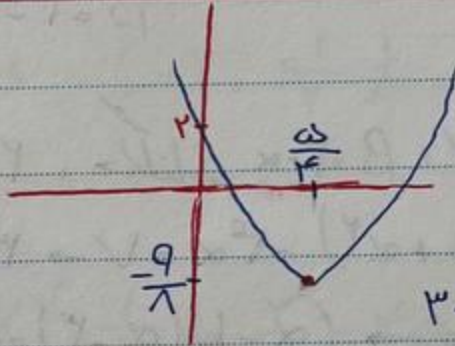


الف) $y = 2x^2 - 5x + 2 \rightarrow \min \left| \frac{5}{4} \right.$

$$\Delta = 25 - 16 = 9$$

از نواحی ۱، ۲، ۳ و ۴ می‌گذرد.

$$\left| \frac{-9}{4} \right.$$

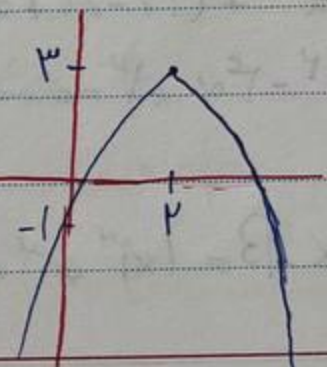


ب) $y = -x^2 + 4x - 1 \rightarrow \max \left| \frac{-b}{-2a} = 2 \right.$

$$\Delta = 16 - (4 \times -1 \times -1) = 12$$

از نواحی ۱ و ۳ عبور می‌کند

$$\left| \frac{-12}{-4} = 3 \right.$$



الف) $\frac{\alpha + \beta}{\alpha - \beta} = \frac{1}{\sqrt{3}}$

$$\Rightarrow x^2 - 2x - 3 = 0 \rightarrow \alpha \cdot \beta = \frac{c}{a} = -3$$

$$\alpha + \beta = \frac{-b}{a} = 1$$

$$\alpha - \beta = \frac{\sqrt{\Delta}}{|a|} \rightarrow \Delta = 1 - (4 \times -3) = 1 + 12 = 13 \rightarrow$$

$$\alpha - \beta = \frac{\sqrt{13}}{1}$$

ب) $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1^2 - (-4) = 5$

ج) $\alpha^3 + \beta^3 = (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) = 1^3 - 3 \times -3 \times 1 = 1 - (-9) = 10$

د) $\alpha^3 - \beta^3 = (\alpha - \beta)(\alpha^2 + \alpha\beta + \beta^2) = \sqrt{13} \times 4 = 4\sqrt{13}$

④ قطع کردن در یک نقطه ← یک ریشه حقیقی متمایز $y = (x-2)(x^2 - ax + a) \rightarrow$

$$\Delta < 0 \rightarrow \Delta = (-a)^2 - 4(1)(a) = a^2 - 4a \rightarrow a^2 - 4a < 0$$

$$\rightarrow a(a-4) < 0 \rightarrow 0 < a < 4 \rightarrow \text{حالت ①}$$

حالت ②: ریشه مشترک $\rightarrow x=2 \Rightarrow 2^2 - 2a + a = 0 \rightarrow a=4$

$$\text{حالت ②} \cup \text{حالت ①} = 0 < a \leq 4$$

⑤ $3x^2 - 12x - a = 0 \rightarrow \underbrace{\alpha + \beta = 4}, \alpha \cdot \beta = \frac{-a}{3}$
 $\beta = 4 - \alpha$

جایگزینی $\beta = 4 - \alpha$ در $2\alpha^2 + \beta^2 - 4\alpha = 7$: $2\alpha^2 + (4 - \alpha)^2 - 4\alpha = 7$

$$2\alpha^2 + (16 - 8\alpha + \alpha^2) - 4\alpha = 7 \rightarrow 3\alpha^2 - 12\alpha + 16 = 7 \rightarrow 3\alpha^2 - 12\alpha + 9 = 0$$

$$\alpha^2 - 4\alpha + 3 = 0 \rightarrow (\alpha - 1)(\alpha - 3) = 0 \rightarrow \alpha = 1$$

$$\alpha = 3 \rightarrow \text{ریشه بزرگتر}$$

$$\alpha \cdot \beta = 1 \times 3 \Rightarrow 3 = \frac{-a}{3} \rightarrow a = \boxed{-9} \Rightarrow \frac{a}{3} = \frac{-9}{3} = \boxed{-3}$$

④ $y = (x-b)^2 + (b-2)$

نسبت به تغییر متغیر x در A, B : $b = \frac{x_A + x_B}{2} = \frac{(2a+3) + (7-2a)}{2} = \frac{10}{2} = 5$

ریشه $S(5, 3)$

جایگزینی A $y = (x-5)^2 + 3$: $a-2 = (2a+3-5)^2 + 3 \rightarrow a-2 = (2a-2)^2 + 3$

$$a-2 = 4(a-1)^2 + 3 \rightarrow a-5 = 4(a-1)^2 \rightarrow a-5 = 4(a^2 - 2a + 1)$$

$$a-5 = 4a^2 - 8a + 4 \rightarrow 4a^2 - 9a + 9 = 0 \rightarrow \Delta = 81 - 144 = -63$$

فاصله از $(0,0)$: $d = \sqrt{0^2 + 28^2} = 28$

$$a = 3$$

محلها A و B صغیر هستند
 $v - 2a > 0 \rightarrow a < 3, 5$
 $2a + 3 > 0 \rightarrow a > -\frac{3}{2}$
 $a - 2 > 0 \rightarrow a > 2$

$$y - 3 = a(u - 5)^2 \xrightarrow{(1,2)} (1-3) = a(1-5)^2 \rightarrow a = \frac{-1}{16}$$

$$(y-3) = -\frac{1}{16}(u-5)^2 \rightarrow (y-3) = -\frac{1}{16}(0-5)^2$$

$$y = -\frac{1}{16} \rightarrow \text{نقطه} = \frac{1}{16}$$

$$4\alpha^2 + 2\alpha - 2\beta = 14 \rightarrow 2\alpha^2 + 2\alpha + 2\beta^2 - 2\beta = 14$$

$$a_1x^2 - a_2x - b = 0 \rightarrow \alpha + \beta = 1, \alpha \cdot \beta = \frac{-b}{a}$$

$$2\alpha^2 + 2\alpha + 2\beta^2 - 2\beta = 14$$

$$2\alpha^2 + 2\alpha + 2\beta^2 - 2\beta = 14 \rightarrow \frac{b}{a} = \frac{-1}{2}$$

$$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta = 1 - \frac{1}{2} = \frac{1}{2}$$

$$\alpha - \beta = \frac{1}{\sqrt{2}}$$

محور $x = 1 + (-\frac{1}{2}) = -\frac{1}{2} \rightarrow$ راس سهمی: $(-\frac{1}{2}, \frac{1}{4})$

معادله سهمی: $y = a(x + \frac{1}{2})^2 - \frac{1}{4}$

از $x=0$: $y = \frac{3}{4} \rightarrow \frac{3}{4} = a(0 + \frac{1}{2})^2 - \frac{1}{4} \rightarrow \frac{3}{4} = \frac{1}{4}a - \frac{1}{4}$

$\rightarrow 2 = \frac{1}{4}a \rightarrow a = \frac{1}{2}$

جایگذاری در معادله: $x=1$: $y = \frac{1}{2}(1 + \frac{1}{2})^2 - \frac{1}{4} \rightarrow y = \frac{1}{2} \times \frac{9}{4} - \frac{1}{4} = \frac{4}{4} = 1$

$$3\alpha^2 + 2\beta^2 = 12\sqrt{2} + 15$$

$$\alpha^2 + 2\alpha^2 + 2\beta^2 = 2(\alpha^2 + \beta^2) + \alpha^2 \rightarrow \alpha^2 + \beta^2 = S^2 - 2P = 34 - 2a$$

$$x^2 + 4x + a = 0 \rightarrow S = -4, P = a$$

$$\alpha = \frac{-4 - \sqrt{16 - 4a}}{2} = \frac{-4 - 2\sqrt{4 - a}}{2} = -2 - \sqrt{4 - a} \rightarrow \beta = -2 + \sqrt{4 - a}$$

$$9 + (4 - a) + 4\sqrt{4 - a} + 2(34 - 2a) = 12\sqrt{2} + 15 \rightarrow 9 - a + 4\sqrt{4 - a} = 12\sqrt{2} + 15 - 20$$

$$\left. \begin{array}{l} \frac{4 - a}{\text{صفر}} + 4\sqrt{4 - a} = 12\sqrt{2} \\ \frac{4 - a}{\text{برابر}} \end{array} \right\} \rightarrow 4\sqrt{4 - a} = 12\sqrt{2} \xrightarrow{\div 4} \sqrt{4 - a} = 3\sqrt{2}$$

$$4 - a = 18 \rightarrow \boxed{a = -14}$$

توان 2 ←

$$\boxed{a = 1}$$

$$39x^2 - (m+14)x + 1 = 0$$

$$\frac{1}{\sqrt{\alpha}} + \frac{1}{\sqrt{\beta}} = \omega \rightarrow \frac{\sqrt{\alpha} + \sqrt{\beta}}{\sqrt{\alpha\beta}} = \omega \rightarrow \sqrt{\alpha} + \sqrt{\beta} = \omega\sqrt{\alpha\beta}$$

$$S + 2\sqrt{P} = \omega\rho \rightarrow S + 2\sqrt{\frac{1}{39}} = \frac{\omega\rho}{39} \rightarrow S = \frac{\omega\rho}{39} - \frac{1}{3} = \frac{13}{39}$$

$$\frac{m+14}{39} = \frac{13}{39} \rightarrow m+14 = 13 \rightarrow m = -1$$

$$mx^2 + 3x + 1 \xrightarrow{m=-1} -x^2 + 3x + 1 \rightarrow \rho = \frac{1}{-1} = -1$$

10

5

10

15

20