آنر پیسا آقاصالحه

۱) الف) $y_v = \frac{-\Delta}{4a} = \frac{-(b^2-4ac)}{12} = \frac{-4+4\cdot 1\cdot(-3)}{12} = -\frac{1}{3}$

$x_v = \frac{-b}{2a} = \frac{2}{6} = \frac{1}{3}$ $\quad S\left(\frac{1}{3}, -\frac{1}{3}\right)$

از ناحیه سوم

ب) $y_v = \frac{-b^2+4ac}{4a} = \frac{-(16-4\cdot(-1)\cdot 3)}{-4} = 4$

$x_v = \frac{-b}{2a} = \frac{-2}{-2} = 1$ $\quad S(1,4)$

ناحیه دوم

۲) الف) $y_v = \frac{-(b^2-4ac)}{4a} = \frac{-(9-4\cdot(-1)\cdot 2)}{8} = -\frac{17}{8}$

$\frac{-b \pm \sqrt{\Delta}}{2a} = \frac{3 \pm 5}{4}$ $\quad \Delta = 9+32 = 41$

$x=2$, $x=-\frac{1}{2}$

$S\left(\frac{3}{4}, -\frac{17}{8}\right)$

ناحیه ① نمی‌گذرد

ناحیه ①، ②، ④ نمی‌گذرد

ب) ③ $y_v = \frac{-(b^2-4ac)}{4a} = \frac{-(16-4\cdot(-1)\cdot(-1))}{-4} = \frac{-12}{-4} = 3$

$x_v = \frac{-b}{2a} = \frac{-4}{-2} = 2$

ریشه‌ها هم علامت $\quad P = \frac{c}{a} = \frac{-1}{-1} = +1 > 0$

ناحیه ④، ③، ①

۳) الف) $\Delta = 1+12 = 13$ $\quad \sqrt{\Delta} = \sqrt{13}$

$|a| = 1$ $\quad$ تفاضل $= \alpha - \beta = \frac{\sqrt{\Delta}}{|a|} = \frac{\sqrt{13}}{1} = \sqrt{13}$

ب)

ج)

د)

$x^2 - x - 3 = 0$ $\quad S = \alpha+\beta = 1$ $\quad P = -3$

$\frac{\alpha+\beta}{\alpha-\beta} = \frac{1}{\sqrt{13}}$

$\alpha^2+\beta^2 = S^2 - 2P = 1^2 - 2(-3) = 1+6 = 7$

$\alpha^3+\beta^3 = S^3 - 3PS = 1^3 - 3(1)(-3) = 1+9 = 10$

$\alpha^3-\beta^3 = (\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2) = \sqrt{13}\times 4 = 4\sqrt{13}$ ($\alpha^2+\beta^2 = 7$, $\alpha\beta = -3$)

$(\alpha-\beta)^2 = \alpha^2+\beta^2-2\alpha\beta \Rightarrow \alpha^2+\beta^2 = 13-6 = 7$

۴)

ریشه اول $\Rightarrow x-2=0$ $\quad x=2$ — جایگذاری

$\Delta = a^2-4a = 0 \Rightarrow a(a-4)=0$ $\quad a=4$ ✓ $\quad a=0$ غ ق ق

$x^2-ax+a \longrightarrow 4-2a+a=0$

$4-a=0$

$a=4$ ✓

$\alpha+\beta=2$

$\alpha\beta=\frac{-a}{3}$

$\alpha^2=(\alpha+\beta)\alpha-\alpha\beta$

$\alpha^2=2\alpha-\alpha\beta$

$\beta=2-\alpha$

$\beta^2=4-4\alpha+\alpha^2$

$\frac{3\alpha^2-12\alpha+9=0}{3}$ ④

$(\alpha-1)(\alpha-3)=0$ ← $\alpha=1$ ← $\boxed{\alpha=3}$

$3(3^2)-12(3)-a=0$

$\boxed{a=-9}$

$A(2a+3,a-2)$ $B(7-2a,a-2)$ $S(b,b-2)$

$b=\frac{(2a+3)+7-2a}{2}=\frac{10}{2}=5$ $S(5,3)$

$\begin{cases}x=0\\y=-2\end{cases}$ طول المسافة $\sqrt{0^2+(-2)^2}=2$ ⑤

$ax^2-ax-b=0$ $\alpha+\beta=1$ $\alpha\beta=\frac{-b}{a}$

$25\beta^2+25\alpha^2-25\beta=17$ → $2\beta^2+\alpha^2-\beta=\frac{17}{25}$ ⑦

$\alpha^2+\beta^2=1-2\alpha\beta$ $(\alpha-\beta)^2=\frac{9}{4}$

$\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$\boxed{|\alpha-\beta|=\frac{3}{2}}$

$x=\frac{1-5}{2}=-2$ $S(-2,-\frac{1}{2})$ $y=a(x+2)^2-\frac{1}{2}$

$a(\frac{3}{2}+2)^2-\frac{1}{2}=0$ $a=\frac{2}{49}$ $\beta=\frac{2}{49}(2)^2-\frac{1}{2}$ $\boxed{\beta=-\frac{13}{91}}$ ①

$\alpha+\beta=-6$ $\alpha=-3-\sqrt{2}$ $a=\alpha\beta=9-2=\underline{7}6$

$\alpha\beta=a$ $\beta=-3+\sqrt{2}$

⑨

$\frac{\alpha+\beta}{\alpha\beta}=5$ ⇐ $\begin{cases}\alpha+\beta=\frac{m+12}{49}\\ \alpha\beta=\frac{1}{49}\end{cases}$ $m+12=-5$ $\boxed{m=-9}$

$\frac{2}{m}=\frac{2}{-9}$

$\boxed{\frac{2}{-9}}$ ↓

⑩