۱-

الف) $(9, x+2y) = (3x-y, -3)$

$9 = 3x - y \qquad 3x - 9 = y \qquad 3x - 9 = \frac{-3-x}{2}$

$-3 = x + 2y \qquad \frac{-3-x}{2} = y \qquad 6x - 18 = -3 - x$

$7x = 14 \rightarrow x = 2$

$3\times 2 - 9 = -3 \qquad \frac{x}{y} = \frac{-2}{3} \qquad y = -3$

ب) $(-1, -3) = \left(\frac{1}{x} - \frac{1}{y}, \frac{5}{x} - \frac{7}{y}\right)$

$\frac{2}{2x} - \frac{1}{2x} = \frac{1}{2x} = -1$

$-1 = \frac{y-x}{xy} \rightarrow -xy = y - x$

$y = -1 \leftarrow \boxed{x = \frac{-1}{2}} \leftarrow 1 = -2x$

$-3 = \frac{5}{x} - \frac{7}{y} \rightarrow -3 = \frac{5y - 7x}{xy} = -3 \rightarrow 5y - 7x = -3xy$

$\leftarrow \therefore \frac{5y - 7x}{3} = -xy$

$3y - 3x = 5y - 7x$

$4x = 2y \rightarrow y = 2x$

$\frac{x}{y} = \frac{\frac{-1}{2}}{\frac{-1}{1}} = \frac{-1}{-2} = \frac{1}{2}$

۲-

$f = \{(a, 2a), (1, a+1), (1, -2), (2, b)\}$

$a + 1 = -2 \rightarrow a = -3$

~~$f(a) + 2f(2)$~~ $= 3f(1)$

$\underset{-6}{2a} + 2b = -6 \qquad 2b = 0 \rightarrow b = 0$

۳-

$f = \{(-1, m^2 - 3m), (3, 5), (-1, -2), (m+1, 9), (2, 4), (m^2 - 2, 4m + 1)\}$

$m^2 - 3m = -2 \rightarrow m(m-3) = -2$

$m^2 - 3m + 2 = 0 \rightarrow (m-2)(m-1)$ ، $m = 2$ ، $m = 1$

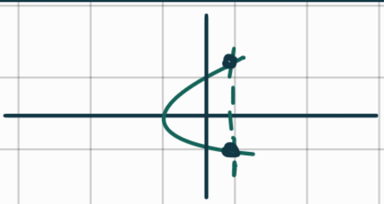
× $m = 1 \rightarrow \{(-1, -2), (3, 5), (-1, -2), (2, 9), (2, 4) \ldots\}$

× $m = 2 \rightarrow \{(-1, -8), (3, 5), (-1, -2) \ldots\}$

(به ازای هیچ مقداری تابع نیست)

۴-

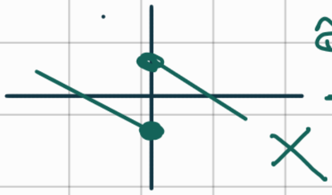
الف)

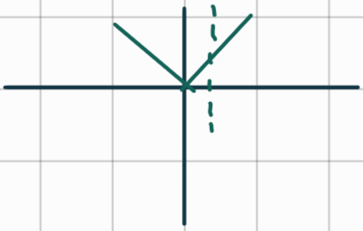


خط موازی محور y ها رسم می کنیم اگر به ازای یک x

دو یا چند تا y داشت تابع نیست!

ب)  ✓

ج)  $\to x=0 \to y$ ۲ دارد ✗

د)  ✓

۵- الف) $y=-\sqrt{x+1}$

$y^2=x+1 \to x=y^2-1$

$x=y_1^2-1$ $\quad$ $y_1^2-1=y_2^2-1$

$x=y_2^2-1$ $\quad$ $y_1^2=y_2^2$

یک x دو y دارد $\quad$ $y_1=\pm y_2 \to$ ✗

ب) $x=\frac{y}{\sqrt{1-y^2}}$

$\left.\begin{array}{l} x=\frac{y_1}{\sqrt{1-y_1^2}} \\ x=\frac{y_2}{\sqrt{1-y_2^2}} \end{array}\right\}$ $\quad \frac{y_1}{\sqrt{1-y_1^2}}=\frac{y_2}{\sqrt{1-y_2^2}}$

$\left(y_1\sqrt{1-y_2^2}\right)^2=\left(y_2\sqrt{1-y_1^2}\right)^2$

$y_1^2(1-y_2^2)=y_2^2(1-y_1^2)$

$y_1^2-\cancel{y_1^2y_2^2}=y_2^2-\cancel{y_1^2y_2^2}$

✗ $y_1^2=y_2^2 \to y_1=\pm y_2$

۶- الف) $|y|=x \to x=1 \to y=\pm 1$ ✗

مثال نقض

ب) $y^3+3y^2+3y+x^3+x=0$ ✓

$\left.\begin{array}{l} y_1^3+3y_1^2+3y_1=-x^3-x \\ y_2^3+3y_2^2+3y_2=-x^3-x \end{array}\right]$ $1+y_1^3+3y_1^2+3y_1=y_2^3+3y_2^2+3y_2+1$

$(y_1+1)^3=(y_2+1)^3$

$y_1+\cancel{1}=y_2+\cancel{1}$

$y_1=y_2$

۷- $f(x)=\frac{x^2+4x+5}{x^2+4x+7}$

$f(\sqrt{3}-2)=\frac{(\sqrt{3}-2)^2+4(\sqrt{3}-2)+5}{(\sqrt{3}-2)^2+4(\sqrt{3}-2)+7}$

۴

$$\frac{3+4-4\sqrt{3}+4\sqrt{3}=8+5}{\frac{3+4-4\sqrt{3}+4\sqrt{3}=8+7}{6}}=\frac{4}{6}=\frac{2}{3}$$

۸-

$(-1, -4)$

$(-1) = (-1)^3 - a - b$

$f(-1) = -1 - a - b$

$-1 - a + b = -4 \rightarrow b - a = -3 \rightarrow \underline{b = -2}$

ضرب

$-4 + 3 + a = 0 \rightarrow \underline{a = 1}$

$y = 3x - 1$
$f(x) = x^3 + x - 2$

$3x - 1 = x^3 + x - 2$
$x^3 - 2x - 1 = 0$
ریشه $\rightarrow -1$

$x^3 - 2x - 1 \div (x+1) = x^2 - x - 1$
$-(x^3 + x^2)$
$-x^2 - 2x - 1$
$-(-x^2 - x)$
$-x - 1$
$-(-x - 1)$
0

$x^2 - x - 1 \rightarrow \Delta = 5$

$$\frac{+1 \pm \sqrt{5}}{2}$$

$$\underbrace{\frac{+1+\sqrt{5}}{2}}_{x_1} + \underbrace{\frac{+1-\sqrt{5}}{2}}_{x_2} = \textcircled{1}$$

۹-

$f = \{(2, a+b), (1, 2a), (-1, a-2b+1)\}$

$y = K \rightarrow$ ثابت
y ها برابر

$a + b = 2a \rightarrow b = a$

$a - 2a + 1 = 2a$

$-a + 1 = 2a \rightarrow 1 = 3a \rightarrow \underline{a = \frac{1}{3} = b}$

۱۰-

$$f(x) = \frac{4x^2 - ax + c + 1}{bx - 3}$$

$y = x \rightarrow$ همانی

$$Fx^2 - dx + c + 1 = x(bx + 3)$$

$$Fx^2 - dx + c + 1 = bx^2 + 3x$$

$$\begin{cases} c = -1 \\ b = 4 \\ d = -3 \end{cases}$$

$$-1 + 4 - 3 = 0$$

کیفیاتی