تکلیف سری ۲۸ ام ۱۸٫۷۵ جامعه‌شناسی...

(۱)

$$f(x)=\begin{cases} x^2+2x & x>a \\ ax-4 & x\le a \end{cases} \Rightarrow a^2+2a=a^2-4 \rightarrow 2a=-4 \Rightarrow \boxed{a=-2}$$ ۳

(۲)

$$f(x)=\frac{x^2+a}{2x-b}, \quad \underbrace{g(x)=2x+b}_{\text{جایگزاری نقطه } (2,3)} \Rightarrow 3=4+b \Rightarrow \boxed{b=-1}$$ ۳

$$\Downarrow$$

$$f(x)=\frac{x^2+a}{2x+1} \xRightarrow[(2,3)]{\text{جایگزاری نقطه}} 3=\frac{4+a}{\underbrace{4+1}_{5}} \Rightarrow 15=4+a \Rightarrow \boxed{a=11}$$

$$f(x)=\frac{x^2+11}{2x+1} \Rightarrow f(1)=\frac{1+11}{2+1} \Rightarrow \frac{12}{3}=\boxed{4}$$ جواب

(۳)

$$f(x)=\frac{4x+1}{2x^2+ax+b}, \quad D_f=R-\{-1,4\} \Rightarrow 2x^2+ax+b=0$$

$x=-1 \Rightarrow 2-a+b=0$

$x=4 \Rightarrow 32+4a+b=0$

$f(1)=?$

$$\begin{cases} -a+b=-2 \\ 4a+b=-32 \end{cases} \times -1 \Rightarrow \begin{array}{l} -a+\cancel{b}=-2 \\ -4a-\cancel{b}=32 \\ \hline -5a=30 \end{array} \Rightarrow \boxed{a=-6}, \boxed{b=-8}$$ ۳

$$f(x)=\frac{4x+1}{2x^2-6x-8} \Rightarrow f(1)=\frac{4+1}{2-6-8}=-\frac{5}{12}$$ جواب

(۴)

$$f(x)=\frac{+x^3+\sqrt{3}}{-4x^2+ax+b} \quad D_f=R-\{-1\} \quad a+b=?\ -8-4=-12$$ جواب

$$\Delta=0 \rightarrow a^2-4(-4)(b)=0 \Rightarrow a^2+16b=0 \Rightarrow \frac{-b}{2a}=-1 \Rightarrow \frac{-a}{-8}=-1 \rightarrow \boxed{a=-8}$$

$$-4(-1)^2+(-8)(-1)+b=0 \Rightarrow -4+8+b=0 \rightarrow \boxed{b=-4}$$ ۳

(۵) ۵٫۷۵

$$f(x)=\frac{2x}{\underline{(x^2+mx+1)}(x-1)} \quad D_f=R-\{1\} \quad \text{حدود } m=? \quad m\in(-2,2)\ \text{II}$$

$$\Delta<0 \rightarrow m^2-4(1)(1)<0 \rightarrow m^2-4<0 \rightarrow \begin{array}{c} -2 \quad 2 \\ \hline + \quad - \quad + \end{array}$$

$x=1$ می‌تواند ریشه مضاعف آن باشد

$(x-1)^2=x^2-2x+1 \rightarrow m=-2$ I

$\text{I} \cup \text{II} \rightarrow [-2,2)$

(۶)

$f(x) = \sqrt{4 - \frac{1}{x^2}}$

$\begin{cases} 4 - \frac{1}{x^2} \geq 0 \\ x^2 \neq 0 \rightarrow x \neq 0 \end{cases}$

$-\frac{1}{2} \quad 0 \quad \frac{1}{2}$

$+ \quad - \quad - \quad +$

$D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

(۷)

$f(x) = \sqrt{mx^2 + 2mx + 1}$ $\qquad m > 0 ,\ \Delta \leq 0$

$(I)\ m > 0 \quad , (II)\ \Delta \leq 0 \Rightarrow 4m^2 - 4(m)(1) \leq 0 \Rightarrow 4m^2 - 4m \leq 0 \Rightarrow 4m(m-1) \leq 0$

$0 \quad 1$

$+ \quad - \quad +$ $\Rightarrow (II)\ [0,1] \rightarrow I \cap II \Rightarrow [0,1] \rightarrow m \in [0,1]$

(۸)

$f(x) = \begin{cases} \frac{4x^2 - 1}{2x - 1} & ; x \neq a \\ 4x + k & ; x = \frac{1}{2} \end{cases}$ $\qquad g(x) = 2x + 1$

$2x - 1 \neq 0 \Rightarrow 2x = 1 \rightarrow \boxed{x = \frac{1}{2}} \rightarrow a = \frac{1}{2}$

$a + k = ?\ \frac{1}{2}$ $\qquad 4(\frac{1}{2}) + k = 2(\frac{1}{2}) + 1 \rightarrow 2 + k = 2 \rightarrow \boxed{k = 0}$

(۹)

$f(x) = \begin{cases} \frac{9x^2 - 4}{3x + 2} & ; x \neq -\frac{2}{3} \\ 3ax + 2 & ; x = -\frac{2}{3} \end{cases}$ $, g(x) = 3x + b$ $\qquad$ جواب $\quad a - b = 3 - (-2) = \boxed{5}$

$x \neq a$

$f(x) = \frac{9x^2 - 4}{3x + 2} = 3x + b \rightarrow \frac{(3x-2)(3x+2)}{3x+2} = 3x + b \Rightarrow 3x - 2 = 3x + b \rightarrow \boxed{b = -2}$

$g(-\frac{2}{3}) = f(-\frac{2}{3}) \Rightarrow 3(-\frac{2}{3})a + 2 = 3(-\frac{2}{3}) - 2 \rightarrow -2a + 2 = -4 \Rightarrow -2a = -6 \Rightarrow \boxed{a = 3}$

(۱۰)

$f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & x \neq 2 \\ 2a^2 + ax & x = 2 \end{cases}$ $\quad , g(x) = x + 2$ به ازای ۱ و ۲- ، a تابع‌های f و g برابرند

$g(2) = f(2) \rightarrow 2a^2 + 2a = 4 \rightarrow 2a^2 + 2a - 4 = 0 \Rightarrow a^2 + a - 2 = 0$ جمع ضرایب مساوی صفر $\rightarrow 1 ,\ -2$