① $f(x) = \begin{cases} x^2+2x \ ; \ x \geq a \\ ax-4 \ ; \ x \leq a \end{cases}$ $\quad a^2+2a = a^2-4$, $\quad a=-2$

② $f(x) = \dfrac{x^2+a}{2x-b}$ ⟵ نقطه تقاطع $(2,3)$

$4+b=3 \Rightarrow \boxed{b=-1}$

$g(x) = 2x+b$ $\quad 4+a = 4+b \Rightarrow 4+a=15 \Rightarrow \boxed{a=11}$

$f(1) = \dfrac{1+11}{2+1} \rightarrow \boxed{4}$

③ $f(x) = \dfrac{4x+1}{2x^2+ax+b}$ $\quad R-\{-1,4\}$ ⟵ دامنه

$a(x+1)(x-4) = a(x^2-3x-4) = 2x^2-6x-8$

$a=-6 . \ b=-8 \rightarrow f(1) = \dfrac{5}{2-6-8} = \dfrac{-5}{12}$

④ $f(x) = \dfrac{x^3-\sqrt{3}}{-4x^2+ax+b}$ $\quad R-\{-1\}$ ⟵ دامنه

$a_4 : (x+1)^2 = -4(x^2+2x+1) \Rightarrow -4x^2-8x-4$

$a=-8 \quad b=-4 \longrightarrow a+b = -8-4 = -12$

⑤ $f(x) = \dfrac{2x}{\underset{1}{\cancel{(x-1)}}\underset{\Delta<0}{(x^2+mx+1)}}$ $\quad R-\{1\}$

$m^2-4<0$

$-2<m<2$

⑥ $f(x) = \sqrt{4-\dfrac{1}{x^2}}$ $\quad x \neq 0 \quad 4-\dfrac{1}{x^2} \geq 0$ $\quad$ کوثر عابدینی ۲۸

$(2-\dfrac{1}{x})(2+\dfrac{1}{x}) > 0$ $\quad \overset{-\frac{1}{2} \quad \frac{1}{2}}{+ \ | \ - \ | \ +}$ $\quad x \leq -\dfrac{1}{2} \ , \ \dfrac{1}{2} \leq x$

$D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

⑦ $f(x) = \sqrt{mx^2+2mx+1}$ $\quad - D_f = R$

$3m+1 \geq 0 \quad -m+1 \geq 0 \rightarrow -\frac{1}{3} \leq m \leq 1$

$m \geq -\frac{1}{3} \qquad m \leq 1$

⑧ $f(x) \begin{cases} \dfrac{\cancel{(2x-1)}(2x+1)}{\cancel{2x-1}} = \dfrac{4x^2-1}{2x-1} \ ; \ x \neq a = \frac{1}{2} \\ 4x+k \ ; \ x = \frac{1}{2} \end{cases}$ $\quad g(x) = 2x+1$

$2+k = 2 \Rightarrow k=0 \quad a+k = \frac{1}{2}+0 = \frac{1}{2}$

⑨ $f(x) = \begin{cases} \dfrac{\cancel{(3x+2)}(3x-2)}{\cancel{3x+2}} = \dfrac{9x^2-4}{3x+2} \ ; \ x \neq -\frac{2}{3} \\ 3ax+2 \ ; \ x = -\frac{2}{3} \end{cases}$ $\quad g(x) = 3x+b$

$\cancel{3x}+b = \cancel{3x}-2 \Rightarrow b=-2 \quad -2a+2 = -2+b = -4 \quad -2a=-6 \Rightarrow a=3$

$a-b = 3-(-2) = 5$

⑩ $f(x) \begin{cases} \dfrac{\cancel{(x-2)}(x+2)}{\cancel{x-2}} = \dfrac{x^2-4}{x-2} \ ; \ x \neq 2 \\ 2a^2+ax \ ; \ x=2 \end{cases}$ $\quad g(x) = x+2$

$2a^2+2a = 4$

$2(a^2+a-2) = 0$

$2(a+2)(a-1) = 0$

$-2 \qquad 1$

$\boxed{a = -2, 1}$