(۱)

$$f(x)=\begin{cases} x^2+2x & x\geq a \xrightarrow[x\to a]{} a^2+2a \\ ax-4 & x<a \xrightarrow[x\to a]{} a^2-4 \end{cases}$$

$a^2+2a=a^2-4$

$2a=-4$

$a=-2$

(۲)

$g(x)=2x+b$

$f(x)=\dfrac{x^2+a}{2x-b}$

$g(2)=3=2(2)+b$

$b=-1$

$f(2)=3=\dfrac{4+a}{\underbrace{4-b}_{4-(-1)=5}} \to 15=4+a$

$\boxed{a=11}$

$f(x)=\dfrac{x^2+11}{2x+1} \to f(1)=\dfrac{1+11}{2+1}=\dfrac{12}{3}=4$

(۳)

$f(x)=\dfrac{4x+1}{2x^2+ax+b} \to 2x^2+ax+b\neq 0$

$x=-1 \to 2(-1)^2+a(-1)+b=0 \to \begin{cases} 2-a+b=0 \\ 32+4a+b=0 \end{cases}$

$x=4 \to 2(4)^2+a(4)+b=0 \to$

$\times - \begin{cases} -2+a-b=0 \\ 32+4a+b=0 \end{cases}$

$30+5a=0$

$30=-5a$

$b=-8 \quad a=-6$

$f(x)=\dfrac{4x+1}{2x^2-6x-8} \to f(1)=\dfrac{4(1)+1}{2(1)^2-6(1)-8}=\dfrac{-5}{12}$

(۴)

$f(x)=\dfrac{x^3-\sqrt{3}}{-4x^2+ax+b} \quad D_f=R-\{-1\} \to -(2x+k)^2=$

$x=-1 \to -(2(-1)+k)^2=0 \quad \boxed{k=2}$

$-4x^2+ax+b \to$

$-(4x^2+4xk+k^2)=-4x^2-4xk-k^2=-4x^2+ax+b$

$k=2 \to -4x^2-8x-4=-4x^2+ax+b$

$b=-4 \to a=-8$

$\boxed{b+a=-12}$

$x = -1 \rightarrow -4x^2 + ax + b = 0 \rightarrow -4(-1)^2 + b = 0 \rightarrow$

$-4 - a + b = 0 \rightarrow b - a = 4$

⑤

$f(x) = \dfrac{2x}{(x-1)(x^2+mx+1)} \qquad D_f = R - \{1\}$

$x = 1$

$x = 1 \rightarrow (x^2 + mx + 1) = 0 \rightarrow 1 + m + 1 = 0 \rightarrow m = -2$ ①

②

$\Delta = b^2 - 4ac \rightarrow \Delta = m^2 - 4(1)(1) = m^2 - 4 < 0$

$m^2 < 4 \rightarrow -2 < m < 2$

②∩① $[-2, 2) \rightarrow$ حدود مقدار m

⑥

$f(x) = \sqrt{4 - \dfrac{1}{x^2}} \qquad 4 - \dfrac{1}{x^2} \geq 0 \rightarrow 4 \geq \dfrac{1}{x^2} \rightarrow x^2 \geq \dfrac{1}{4}$

$x^2 \neq 0 \rightarrow x \neq 0 \qquad x \geq \frac{1}{2}$ یا $x \leq -\frac{1}{2}$

$D_f = (-\infty, -\frac{1}{2}] \cup [\frac{1}{2}, +\infty)$

⑦

$f(x) = \sqrt{mx^2 + 2mx + 1} \qquad mx^2 + 2mx + 1 \geq 0$ اگر ضریب x^2 مثبت باشد سهمی دهانش به بالا و مثبت می‌شود

$m \geq 0$

$\Delta \leq 0 \rightarrow (2m)^2 - 4(m)(1) \leq 0 \rightarrow 4m^2 - 4m \leq 0$

$4m(m-1) \leq 0$

	0		1	
+		−		+

$m = 0 \quad m = 1 \rightarrow [0, 1]$

/ /

8

$$f(x)=\begin{cases}\dfrac{4x^2-1}{2x-1} & x\neq a \\ 4x+k & x=\dfrac{1}{2}\end{cases}$$

$\to$ صفر به ازای a برابر 0 می‌شود

$2x-1=2a-1=0 \to a=\frac{1}{2}$

$g(x)=2x+1$

$4x+k=2x+1 \qquad \boxed{a+k=\frac{1}{2}}$

$2+k=2$

$\boxed{k=0}$

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$$f(x)=\begin{cases}\dfrac{9x^2-4}{3x+2} & x\neq -\dfrac{2}{3} \\ 3ax+2 & x=-\dfrac{2}{3}\end{cases}$$

$\dfrac{9x^2-4}{3x+2}=3x+b \to (3x+b)(3x+2)=9x^2-4$

$(3x+b)(3x+2)=(3x-2)(3x+2)$

$b=-2$

$g(x)=3x+b \to x=-\frac{2}{3} \to 3ax+2=3x+b$

$3(-\frac{2}{3})a+2=3(-\frac{2}{3})+b \to$

$\boxed{a-b=3-(-2)=5}$

$-2a+2=-2-2$

$-2a=-6$

$\boxed{a=3}$

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$$f(x)=\begin{cases}\dfrac{x^2-4}{x-2} & x\neq 2 \\ 2a^2+ax & x=2\end{cases} \qquad g(x)=x+2$$

$2a^2+ax=x+2 \xrightarrow[x=2]{} 2a^2+2a=4 \to 2a^2+2a-4=0$

$\Delta=b^2-4ac \to 4-4(2)(-4)=36$

$a=\dfrac{-b\pm\sqrt{\Delta}}{2a} \to \dfrac{-2\pm\sqrt{36}}{4} \to \boxed{x_1=-2}, \ \boxed{x_2=1}$