

الف) $y^3 - x = 2x^2 \rightarrow y^3 = 2x^2 + x \Rightarrow y_1^3 = y_2^3 \rightarrow y_1 = y_2$ ✓ تابع هست

ب) $|y| + x^2 = x + 3 \rightarrow |y| = x + 3 - x^2 \xrightarrow{\text{اگر } x=3} |y| = \pm 3$ ✗ تابع نیست

ج) $\sqrt{x-4} + |y-2| = 0 \rightarrow \sqrt{x-4} = 0 \rightarrow x=4$ و $|y-2| = 0 \rightarrow y=2$ } نقطه $(4, 2)$ ✓ تابع هست

د) $x = \cos y \xrightarrow{\text{در جمله ناخفیه}} x=0 \rightarrow y = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$ ✗ تابع نیست

الف) $y = \frac{x+4}{x^2-4x} \rightarrow x(x-4) = 0 \rightarrow D_f = \mathbb{R} - \{0, 4\}$ در همه کسری ها خارج از پرشده / دلتا گرفته شده

ب) $y = \frac{x+4}{x^2-4x+16} \rightarrow \Delta = 16 - 64 = -48 < 0$ خارج از پرشده ندارد $\rightarrow D_f = \mathbb{R}$

ج) $y = \frac{x+4}{x^2+x+4} \rightarrow \Delta = 1 - 16 = -15 < 0$ خارج از پرشده ندارد $\rightarrow D_f = \mathbb{R}$

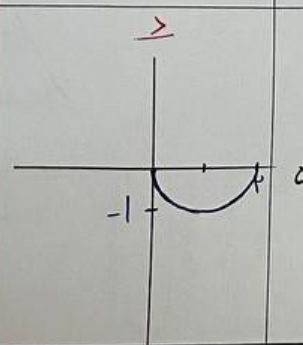
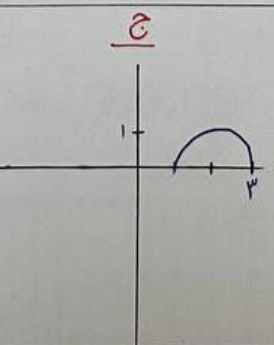
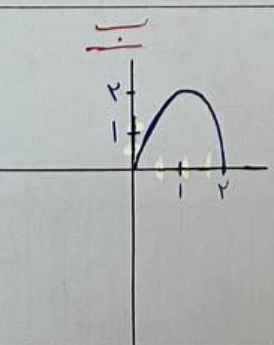
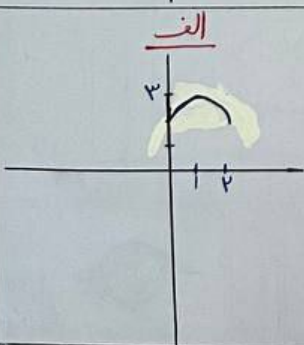
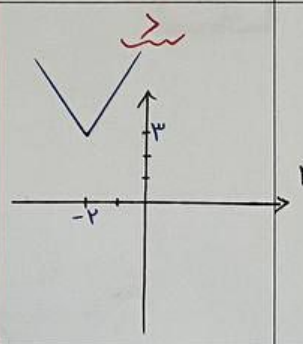
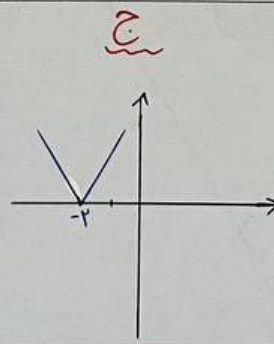
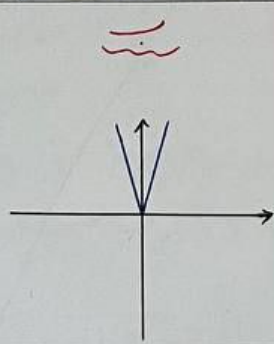
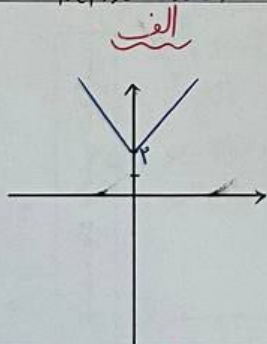
د) $y = \frac{x+4}{x^2-4x^2} \rightarrow x^2(x^2-4) = x^2(x-2)(x+2) = 0 \rightarrow D_f = \mathbb{R} - \{0, -2, 2\}$

الف) $y = \sqrt{4-x} + \sqrt{x-2} \rightarrow \begin{cases} 4-x \geq 0 \Rightarrow x \leq 4 \\ x-2 \geq 0 \Rightarrow x \geq 2 \end{cases} \rightarrow D_f = [2, 4]$

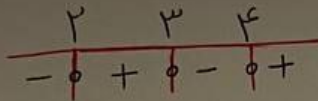
ب) $y = \frac{3x+1}{x^2+1} \rightarrow x^2+1 \neq 0$ همیشه مثبت $\rightarrow D_f = \mathbb{R}$

ج) $y = \frac{\sqrt{x-2}}{x-4} \rightarrow \begin{cases} x-2 \geq 0 \Rightarrow x \geq 2 \\ x-4 \neq 0 \Rightarrow x \neq 4 \end{cases} \rightarrow D_f = [2, 4) \cup (4, +\infty)$

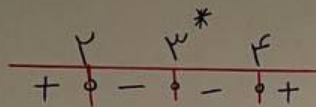
د) $y = \frac{2x+3}{|x|+x^2} \rightarrow x \neq 0 \Rightarrow D_f = \mathbb{R} - \{0\}$ یا $(-\infty, 0) \cup (0, +\infty)$



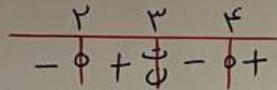
الف) $y = (x-2)(x-3)(x-4)$



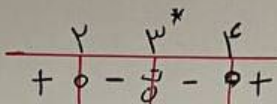
ب) $y = (x-2)(x-3)^2(x-4)$



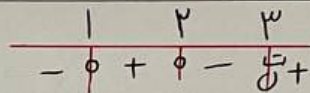
الف) $y = \frac{(x-2)(x-4)}{x-3}$



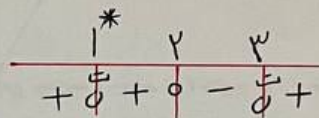
ب) $y = \frac{(x-2)(x-4)}{(x-3)^2}$



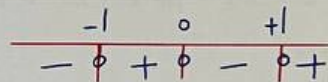
الف) $y = \frac{x^3 - 3x^2 + 2}{x-3} \Rightarrow \frac{(x-1)(x-2)}{x-3}$



ب) $y = \frac{x^3 - 3x^2 + 2}{x^2 - 4x + 3} \Rightarrow \frac{(x-1)(x-2)}{(x-1)(x-3)}$



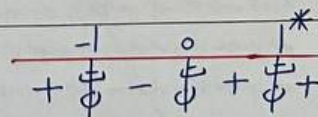
الف) $y = \frac{x^4 - x^2}{x^2 + x + 2} = \frac{x^2(x^2 - 1)}{x^2 + x + 2} = \frac{x^2(x-1)(x+1)}{x^2 + x + 2}$



$\Delta = 1^2 - 4 = -3$ (دیسکریمینانت منفی)

ب) $y = \frac{x^3 + 3x^2 + 4}{x^2 + x + 3}$ هم صورت هم مخرج ریشه حقیقی ندارند پس جواب همیشه مثبت است.

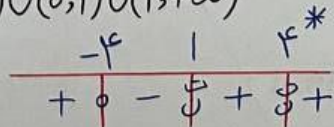
الف) $y = \sqrt{\frac{x^3 - 1}{x^3 - x}} = \frac{(x-1)(x^2 + x + 1)}{x(x+1)(x-1)} \geq 0$



ریشه ها: 1 و 0 و -1

$D_f = (-\infty, -1) \cup (0, 1) \cup (1, +\infty)$

ب) $y = \sqrt{\frac{x^2 - 16}{x^2 - 5x + 4}} \Rightarrow \frac{(x-4)(x+4)}{(x-1)(x-4)} \geq 0$



$D_f = (-\infty, -4] \cup (1, 4) \cup (4, +\infty)$