

الف) $y^3 - x = 2x^2$ تغییر متغیر $\begin{cases} y_1^3 = 2x^2 + x \\ y_2^3 = 2x^2 + x \end{cases} \rightsquigarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2$ شرط ریاضی: $\begin{cases} y_1 = y_2 \text{ if } (x, y) \in F \\ y_1 \neq y_2 \text{ if } (x, y) \notin F \end{cases}$ (تابع هست)

ب) $|y| + x^2 = x + 2$ مثال نقض $x=0$ $|y| + 0 = 0 + 2 \Rightarrow |y| = 2 \Rightarrow y = \pm 2$ (تابع نیست)

ج) $\sqrt{x-4} + |y-2| = 0$ [4] (تابع هست) * در عبارت جمله مثبت حاصل مثل منفرات بین خود را باید در حاصل مثل منفرات باشد.

د) $x = \cos y$ مثال نقض $x=0$ $\cos y = 0 \rightsquigarrow y = \frac{\pi}{2}, \frac{3\pi}{2}, \dots$ (تابع نیست)

الف) $y = \frac{x+4}{x^2-4x}$ ریشه یابی $x^2-4x=0 \Rightarrow x(x-4)=0 \Rightarrow \begin{cases} x=0 \\ x=4 \end{cases}$ $\langle D_f = \mathbb{R} - \{0, 4\} \rangle$

ب) $y = \frac{x+4}{x^2-7x+14}$ ریشه یابی $x^2-7x+14=0 \rightarrow \Delta = b^2-4ac = 49 - (4)(14) = -15$ $\langle D_f = \mathbb{R} \rangle$

ج) $y = \frac{x+4}{x^2+x+4}$ ریشه یابی $x^2+x+4=0 \rightarrow \Delta = b^2-4ac = 1 - (4)(4) = -15$ $\langle D_f \in \mathbb{R} \rangle$

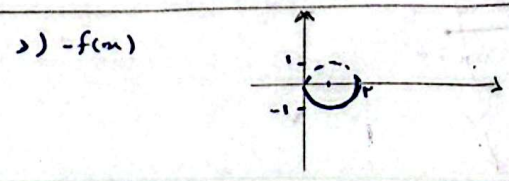
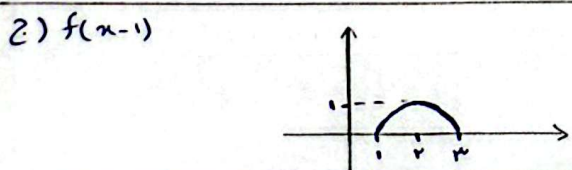
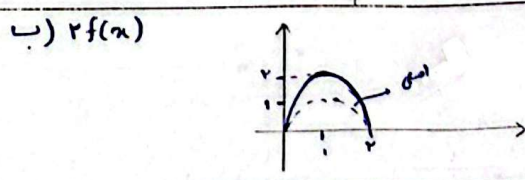
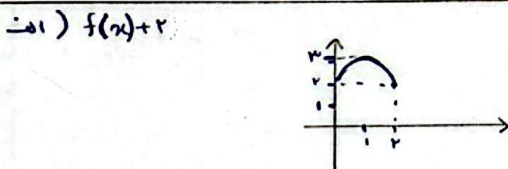
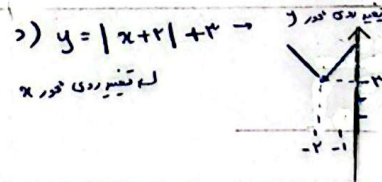
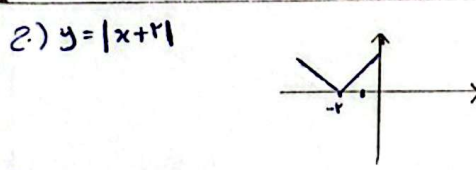
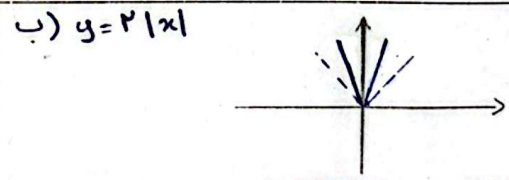
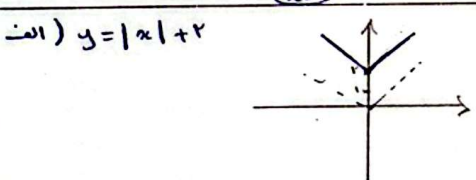
د) $y = \frac{x+4}{x^2-4x^2}$ ریشه یابی $x^2-4x^2=0 \rightarrow x^2(x^2-4)=0 \Rightarrow \begin{cases} x=0 \\ x=2 \\ x=-2 \end{cases}$ $\langle D_f = \mathbb{R} - \{0, \pm 2\} \rangle$

الف) $y = \sqrt{4-x} + \sqrt{x-2}$ مثال نقض $4 \geq x \cap x \geq 2 = 2 \leq x \leq 4 \rightarrow D_f = [2, 4]$

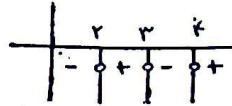
ب) $y = \frac{3x+1}{x^2+1}$ ریشه یابی $x^2+1=0 \rightarrow x^2=-1$ $D_f = \mathbb{R}$

ج) $y = \frac{\sqrt{x-2}}{x-4}$ ریشه یابی $x-4=0 \rightarrow x=4$ $D_f = [2, +\infty) - \{4\}$

د) $y = \frac{2x+3}{|x|+x^2}$ ریشه یابی $|x|+x^2=0$ $D_f = \mathbb{R} - \{0\}$

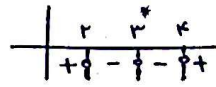


الف) $y = (x-2)(x-3)(x-4)$
 $\begin{matrix} x=2 & x=3 & x=4 \\ \text{---} & \text{---} & \text{---} \end{matrix}$

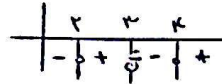


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ب) $y = (x-2)(x-3)(x-4)$
 $\begin{matrix} x=2 & x=3 & x=4 \\ \text{---} & \text{---} & \text{---} \end{matrix}$

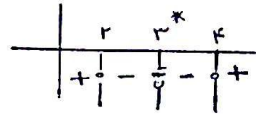


الف) $y = \frac{(x-2)(x-4)}{x-3}$

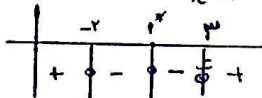


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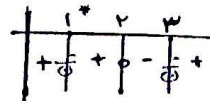
ب) $y = \frac{(x-2)(x-4)}{(x-3)^2}$



الف) $y = \frac{x^3 - 3x^2 + 2}{x^3 - 4x^2 + 4x} = \frac{(x-1)(x-2)}{(x-1)(x-2)} = 1$
 $\begin{matrix} x=1 & x=2 \\ \text{---} & \text{---} \end{matrix}$

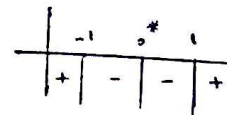


ب) $y = \frac{x^3 - 3x^2 + 2}{x^3 - 4x^2 + 4x} = \frac{(x-1)(x-2)}{(x-1)(x-2)} = 1$
 $\begin{matrix} x=1 & x=2 \\ \text{---} & \text{---} \end{matrix}$



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الف) $y = \frac{x^3 - x^2}{x^3 + x^2 + x} = \frac{x^2(x-1)}{x^2(x+1)} = \frac{x-1}{x+1}$
 $\begin{matrix} x=0 & x=1 & x=-1 \\ \text{---} & \text{---} & \text{---} \end{matrix}$



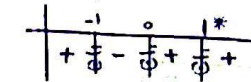
$\Delta = b^2 - 4ac = 1 - 4 = -3 < 0$
 مخرج همیشه ندارد، همواره مثبت

ب) $y = \frac{x^3 + 3x^2 + 4}{x^3 + 2x^2 + 3}$

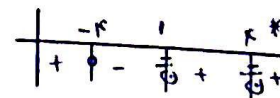
صورت: $\Delta = b^2 - 4ac = 9 - (4 \times 3) = -3 < 0$ همواره مثبت
 مخرج: $\Delta = b^2 - 4ac = 4 - (1 \times 3) = 1 < 0$ همواره مثبت
 همواره مثبت و همواره مثبت، مخرج همیشه دارد

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الف) $y = \frac{x^3 - 1}{x^3 - x} = \frac{(x-1)(x^2+x+1)}{x(x-1)(x+1)} = \frac{x^2+x+1}{x(x+1)}$
 $\begin{matrix} x=0 & x=1 & x=-1 \\ \text{---} & \text{---} & \text{---} \end{matrix}$
 $D_f = (-\infty, -1) \cup (0, 1) \cup (1, +\infty)$



ب) $y = \frac{x^3 - 14}{x^3 - 6x^2 + 4x} = \frac{(x-2)(x+1)}{(x-1)(x-4)}$
 $\begin{matrix} x=1 & x=2 & x=4 \\ \text{---} & \text{---} & \text{---} \end{matrix}$



$D_f = (-\infty, -1) \cup (1, 2) \cup (4, +\infty)$

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