١٨/٥

Subject: سارا مرزبری — تمرینات ۶ — Year: — Month: — Day:

① ۲

الف) $y = \frac{x+4}{4x^3 - 20x^2 + 31x - 15} = \frac{x+4}{(x-1)(4x^2-16x+15)} = \frac{x+4}{(x-1)(2x-5)(2x-3)}$

$4x^3 - 4x^2 + 31x - 15 \;\big|\; x-1 \;\to\; 4x^2 - 16x + 15$, remainder 0

$\to (x-1)(2x-5)(2x-3) \neq 0 \to D_f = \mathbb{R} - \{1, \frac{5}{2}, \frac{3}{2}\}$

$x=1$, $2x=5 \Rightarrow x=\frac{5}{2}$, $2x=3 \Rightarrow x=\frac{3}{2}$

ب) $y = \frac{x+3}{3x^3 - x^2 - 8x - 4} = \frac{x+4}{(x+1)(3x^2-4x-4)} = \ldots \to D_f = \mathbb{R} - \{-1, 2, -\frac{2}{3}\}$

$3x^3 - x^2 - 8x - 4 \;\big|\; x+1 \;\to\; 3x^2 - 4x - 4$, remainder 0

$3x^2 - 4x - 4 = 0 \to x^2 - 4x - 12 = 0 \to (x-6)(x+2) = 0$

$\to x = \frac{6}{3} = 2$ ، $x = -\frac{2}{3}$

② ۱/۷۵ — ۰/۷۵

الف) $y = \frac{x+1}{x - \sqrt{3-2x}} \to 3 - 2x \geq 0 \to 3 \geq 2x \to \frac{3}{2} \geq x$

$\to x - \sqrt{3-2x} \neq 0 \to x \neq \sqrt{3-2x} \to x^2 \neq 3 - 2x \to x^2 + 2x - 3 \neq 0$

$\to (x-1)(x+3) \neq 0 \to x \neq 1, x \neq -3$

$\to D_f = (-\infty, \frac{3}{2}] - \{1, -3\}$ $x = -3 \quad y = \frac{-2}{-6}$

ب) $y = \frac{x+2}{x - \sqrt{3x-2}} \to 3x - 2 \geq 0 \to 3x \geq 2 \to x \geq \frac{2}{3}$

۱

$\to x - \sqrt{3x-2} \neq 0 \to x \neq \sqrt{3x-2} \to x^2 \neq 3x - 2 \to x^2 - 3x + 2 \neq 0$

$\to (x-2)(x-1) \neq 0 \to x \neq 2 ، x \neq 1$

$\to D_f = [\frac{2}{3}, +\infty) - \{2, 1\}$

③ ۲

الف) $y = \frac{\sin x}{2\cos x - 1} \to 2\cos x - 1 \neq 0 \to 2\cos x \neq 1 \to \cos x \neq \frac{1}{2} \to D_f = \mathbb{R} - \{2k\pi \pm \frac{\pi}{3}\}$

ب) $y = \frac{\cos x + 1}{2\sin x + 1} \to 2\sin x + 1 \neq 0 \to 2\sin x \neq -1 \to \sin x \neq -\frac{1}{2} \to D_f = \mathbb{R} - \{2k\pi - \frac{\pi}{6}, 2k\pi + \pi + \frac{\pi}{6}\}$

ج) $y = \frac{\tan x - 2}{\cot x - 1} \to \tan x \neq$ تعریف نشده $\to \mathbb{R} - \{2k\pi \pm \frac{\pi}{2}\}$

$\to \cot x \neq$ تعریف نشده $\to \mathbb{R} - \{k\pi\}$

$\to \cot x - 1 \neq 0 \to \cot x \neq 1 \to \mathbb{R} - \{k\pi + \frac{\pi}{4}\}$

$D_f = \mathbb{R} - \{\frac{k\pi}{2}, k\pi + \frac{\pi}{4}\}$

د) $y = \frac{\sin x + 1}{4\sin^2 x - 3} \to 4\sin^2 x - 3 \neq 0 \to 4\sin^2 x \neq 3 \to \sin^2 x \neq \frac{3}{4} \to \sin x \neq \pm\frac{\sqrt{3}}{2}$

$D_f = \mathbb{R}\{k\pi \pm \frac{\pi}{3}\}$

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الف) $x^2-5x+6>0 \to (x-3)(x-2)>0$ — $\overset{2\quad 3}{+\;\;-\;\;+}$ $\to D_f=(-\infty,2)\cup(3,+\infty)$ **۲**

ب) $x^2-6x+5<0 \to (x-5)(x-1)<0$ — $\overset{1\quad 5}{+\;\;-\;\;+}$ $\to D_f=(1,5)$

ج) $x^2+6x+5\geq 0 \to (x+5)(x+1)\geq 0$ — $\overset{-5\quad -1}{+\;\;-\;\;+}$ $\to D_f=(-\infty,-5]\cup[-1,+\infty)$

د) $x^2-7x+6\leq 0 \to (x-6)(x-1)\leq 0$ — $\overset{1\quad 6}{+\;\;-\;\;+}$ $\to D_f=[1,6]$

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الف) $\frac{x^2-3x+2}{x-5}<0 \to \frac{(x-2)(x-1)}{x-5}<0$ — $\overset{1\quad 2\quad 5}{-\;\;+\;\;-\;\;+}$ $\to D_f=(-\infty,1)\cup(2,5)$ **۱٫۷۵**

-1
1
5

ب) $\frac{x^2-1}{x^2-6x+5}\geq 0 \to \frac{(x+1)(x-1)}{(x-5)(x-1)}\geq 0$ — $\overset{-1\quad 1\quad 5}{+\;\;-\;\;-\;\;+}$ $\to D_f=(-\infty,1]\cup[5,+\infty)$ **۰٫۷۵**

۶

الف) $y=\sqrt{\frac{x^2-4x+3}{x^3-1}} \to \frac{(x-3)(x-1)}{(x-1)(x^2+x+1)}\geq 0$ — ~~$\overset{1\quad 3}{+\;\;-\;\;+}$~~ $\to D_f=$ ~~$(-\infty,1)\cup[3,+\infty)$~~ **۱**

ب) $y=\sqrt[3]{\frac{x^2-4x+3}{x^2-1}} \to x^2-1\neq 0 \to (x+1)(x-1)\neq 0 \to x\neq\pm 1 \to D_f=\mathbb{R}-\{\pm 1\}$ ✓

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الف) $y=\log(x^2-3x) \to x^2-3x>0 \to x(x-3)>0$ — $\overset{0\quad 3}{+\;\;-\;\;+}$ $\to D_f=(-\infty,0)\cup(3,+\infty)$ **۲**

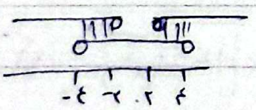
ب) $y=\log_{|x|-2}(16-x^2)$

$\to 16-x^2>0 \to (4-x)(4+x)>0$ — $\overset{-4\quad 4}{-\;\;+\;\;-}$

$\to |x|-2>0 \to |x|>2 \to x>2$ یا $x<-2$

$\to |x|-2\neq 1 \to |x|\neq 3 \to x\neq\pm 3$

$D_f=(-4,-2)\cup(2,4)-\{\pm 3\}$



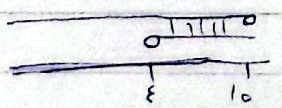
ج) $y=\log_{10-x}\frac{x^2-7x+12}{x-3}$

$\to \frac{x^2-7x+12}{x-3}>0 \to \frac{(x-3)(x-4)}{x-3}>0$ — $\overset{3\quad 4}{-\;\;-\;\;+}$

$\to 10-x>0 \to 10>x$

$\to 10-x\neq 1 \to 9\neq x$

$D_f=(4,10)-\{9\}$

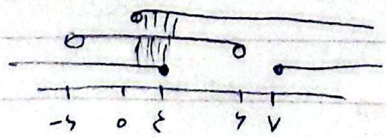


د) $y=\sqrt{x^2-11x+28}+\log_x\sqrt{36-x^2}$

$\to x^2-11x+28\geq 0 \to (x-4)(x-7)\geq 0$ — $\overset{4\quad 7}{+\;\;-\;\;+}$

$\to \sqrt{36-x^2}>0 \to 36-x^2>0 \to (6-x)(6+x)>0$ — $\overset{-6\quad 6}{-\;\;+\;\;-}$

$\to x>0$ و $x\neq 1$



$D_f=(0,4]-\{1\}$

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①

$y = \frac{x^2 - x}{x^2 + x}$ تعیین علامت:

	-1		0		1	
$x^2 - x$	+	+	0	-	0	+
$x^2 + x$	+ 0	-	0	+		+
$y = f(x)$	+ ∄	-	∄	-	0	+

↓

$y = \frac{x(x-1)}{x(x+1)}$

نتیجه:

-1		0		1	
+ ∄	-	∄	-	0	+

②

$y = \sqrt{\frac{x - f(x)}{f(x)}}$

$\frac{x - f(x)}{f(x)} \geq 0 \rightarrow x - f(x) = 0 \rightarrow x = f(x) \rightarrow x = ۳, -۱$

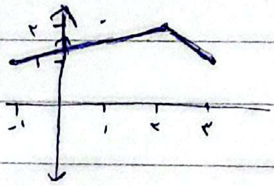
$\searrow f(x) = 0 \rightarrow x = ۲, -۲$

$f(x) \neq 0$

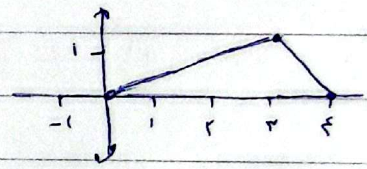
~~-۲   -۱   ۲   ۳ : + ∄ - 0 + ∄ - 0 +~~ → ~~$D_f = (-\infty, -۲) \cup [-۱, ۲) \cup [۳, +\infty)$~~

⑩

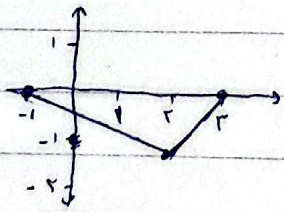
الف) $f(x) + 1$



ب) $f(x-1)$



ج) $-f(x)$



د) $f(x+1) + ۲$