

الف) $y = \frac{x+4}{x^3 - 10x^2 + 31x - 15}$ $\rightarrow x^3 - 10x^2 + 31x - 15 = 0 \rightarrow x = 1$
 $\rightarrow x^3 - 10x^2 + 31x - 15 = (x-1)(x-6)(x-10)$
 $D_f = \mathbb{R} - \{1, \frac{3}{2}, \frac{5}{2}\}$
 $x = \frac{4}{6} = \frac{2}{3}$
 $x = \frac{10}{6} = \frac{5}{3}$

ب) $y = \frac{x+3}{x^3 - x^2 - 11x - 4}$ $\rightarrow x^3 - x^2 - 11x - 4 = 0 \rightarrow x = -1$
 $\rightarrow x^3 - x^2 - 11x - 4 = (x+1)(x-4)(x+2)$
 $D_f = \mathbb{R} - \{-1, -\frac{2}{3}, 2\}$
 $x = \frac{4}{3} = \frac{4}{3}$
 $x = -\frac{2}{3}$

الف) $y = \frac{x+1}{x - \sqrt{3-2x}}$ $\rightarrow x - \sqrt{3-2x} \neq 0 \rightarrow x \neq \sqrt{3-2x}$
 $\rightarrow x^2 \neq 3-2x \rightarrow x^2 + 2x - 3 \neq 0 \rightarrow (x+3)(x-1) \rightarrow x = -3$
 $x = 1$
 $3-2x \geq 0 \rightarrow x \leq \frac{3}{2}$
 $D_f = (-\infty, -3) \cup (-3, 1) \cup (1, \frac{3}{2}]$

ب) $y = \frac{x+2}{x - \sqrt{3x-2}}$ $\rightarrow x - \sqrt{3x-2} \neq 0 \rightarrow x \neq \sqrt{3x-2}$
 $\rightarrow x^2 \neq 3x-2 \rightarrow x^2 - 3x + 2 \neq 0 \rightarrow (x-1)(x-2) \rightarrow x \neq 1$
 $x \neq 2$
 $3x-2 \geq 0 \rightarrow x \geq \frac{2}{3}$
 $D_f = \{x \in \mathbb{R} \mid x \geq \frac{2}{3}, x \neq 1, x \neq 2\}$

الف) $y = \frac{\sin x}{2\cos x - 1}$ $\rightarrow 2\cos x - 1 \neq 0 \rightarrow \cos x \neq \frac{1}{2}$
 $\frac{\pi}{3} \leftarrow \cos x = \frac{1}{2}$
 $\frac{5\pi}{3} \leftarrow \cos x = \frac{1}{2}$
 $D_f = \mathbb{R} - \{2k\pi \pm \frac{\pi}{3}\}$

ب) $y = \frac{\cos x + 1}{2\sin x + 1}$ $\rightarrow 2\sin x + 1 \neq 0 \rightarrow \sin x \neq -\frac{1}{2}$
 $\frac{7\pi}{6} \leftarrow \sin x = -\frac{1}{2}$
 $\frac{11\pi}{6} \leftarrow \sin x = -\frac{1}{2}$
 $D_f = \mathbb{R} - \{2k\pi + \frac{7\pi}{6}, 2k\pi + \frac{11\pi}{6}\}$

ج) $y = \frac{\tan x + 1}{\cot x - 1}$ $\rightarrow \tan x \neq \frac{1}{\cot x - 1}$
 $\rightarrow \tan x \neq \frac{1}{\frac{1}{\tan x} - 1} \rightarrow \tan x \neq \frac{\tan x}{1 - \tan x} \rightarrow \tan x \neq \frac{\tan x}{1 - \tan x}$
 $\rightarrow \tan x \neq \frac{\tan x}{1 - \tan x} \rightarrow \tan x \neq \frac{\tan x}{1 - \tan x}$
 $\rightarrow \tan x \neq \frac{\tan x}{1 - \tan x} \rightarrow \tan x \neq \frac{\tan x}{1 - \tan x}$
 $D_f = \mathbb{R} - \{k\pi, k\pi + \frac{\pi}{4}, k\pi + \frac{3\pi}{4}\}$

د) $y = \frac{\sin x + 1}{2\sin^2 x - 3}$ $\rightarrow 2\sin^2 x - 3 \neq 0 \rightarrow \sin^2 x \neq \frac{3}{2}$
 $\sin x \neq \frac{\sqrt{3}}{2} \rightarrow k\pi + \frac{\pi}{3}$
 $\sin x \neq -\frac{\sqrt{3}}{2} \rightarrow k\pi + \frac{2\pi}{3}$
 $D_f = \mathbb{R} - \{k\pi + \frac{\pi}{3}, k\pi + \frac{2\pi}{3}\}$

الف) $x^2 - 5x + 4 > 0 \rightarrow \frac{4}{1} - \frac{3}{1} \rightarrow D_f = (-\infty, 2) \cup (3, +\infty)$
 $(x-2)(x-3)$

ب) $x^2 - 4x + 5 < 0 \rightarrow \frac{1}{1} - \frac{4}{1} \rightarrow D_f = (1, 5)$
 $(x-1)(x-5)$

ج) $x^2 + 4x + 5 \geq 0 \rightarrow \frac{-5}{1} - \frac{-1}{1} \rightarrow D_f = (-\infty, -5] \cup [-1, +\infty)$
 $(x+1)(x+5)$

د) $x^2 - \sqrt{x} + 4 \leq 0 \rightarrow \frac{4}{1} - \frac{1}{1} \rightarrow D_f = [1, 4]$
 $(x-1)(x-4)$

$D_f = [1, 4]$

الف) $\frac{x^2 - 3x + 2}{x - 5} < 0 \rightarrow (x-1)(x-2) \rightarrow \frac{1}{-} \frac{1}{+} \frac{2}{-} \frac{5}{+} \left\} D_f = (-\infty, 1) \cup (2, 5) \right.$ (5)

ب) $\frac{x^2 - 1}{x^2 - 4x + 5} \geq 0 \rightarrow (x-1)(x+1) \rightarrow \frac{-1}{+} \frac{1}{-} \frac{5}{-} \frac{5}{+} \rightarrow D_f = (-\infty, -1] \cup (5, +\infty)$

الف) $y = \sqrt{\frac{x^2 - 4x + 3}{x^3 - 1}} \rightarrow \frac{(x-1)(x-3)}{(x-1)(x^2+x+1)} \rightarrow \frac{1}{-} \frac{3}{-} \frac{1}{+} \left\} D_f = [3, +\infty) \right.$ (9)

ب) $y = \sqrt[3]{\frac{x^2 - 4x + 3}{x^2 - 1}} \rightarrow (x+1)(x-1) \quad D_f = \mathbb{R} - \{-1, 1\}$

الف) $y = \log(x^2 - 3x) \rightarrow x^2 - 3x > 0 \rightarrow x < 0 \rightarrow x > 3 \left\} D_f = (-\infty, 0) \cup (3, +\infty) \right.$ (7)

ب) $y = \log \frac{19 - x^2}{|x| - 2} \rightarrow 19 - x^2 > 0 \rightarrow x^2 < 19 \rightarrow -4 < x < 4$
 $|x| - 2 > 0 \rightarrow |x| > 2 \rightarrow x > 2 \vee x < -2$
 $|x| \neq 3 \rightarrow x \neq \pm 3$

ج) $y = \log \frac{x^2 - \sqrt{x+11}}{x-3} \rightarrow \frac{(x-3)(x-4)}{x-3} \rightarrow \frac{1}{-} \frac{4}{-} \frac{3}{+} \left\} D_f = (4, 9) \cup (9, 10) \right.$
 $10 - x > 0 \rightarrow x < 10$
 $10 - x \neq 1 \rightarrow x \neq 9$

د) $y = \sqrt{x^2 - 11x + 28} + \log \sqrt{39 - x^2} \rightarrow (4-x)(4+x) \rightarrow -4 < x < 4$
 $\frac{4}{+} \frac{4}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \left\} D = (0, 1) \cup (1, 4] \right.$
 $\frac{4}{+} \frac{4}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \frac{1}{-} \frac{1}{+} \rightarrow (-\infty, 4] \cup [7, +\infty)$
 $x \neq 4 \quad x \neq 7$

	-1	0	1		-1	0*	1	
$x^2 - x$	-	+	0	-	+	0	-	+
$x^2 + x$	-	0	-	0	+	+	+	+
$\frac{x^2 - x}{x^2 + x} = y$	+	0	-	0	-	0	+	+

الف) $y = \sqrt{\frac{x - f(x)}{f(x)}} \rightarrow \frac{x - f(x)}{f(x)} \geq 0 \rightarrow x \geq f(x) \leftarrow f(x) > 0$
 $\rightarrow x \leq f(x) \leftarrow f(x) < 0$
 $\rightarrow x = -1, x = 3$
 $\rightarrow x \neq \pm 2$
 $D_f = (-1, 2) \cup (3, +\infty)$ (9)

