تکلیف شماره ۵
کیمیا مرادی

1-

الف) $y = \dfrac{x+4}{4x^3 - 20x^2 + 31x - 15}$ →

$x = 1$ ، $x-1$ مقسوم‌علیه

$$\begin{array}{l} 4x^3 - 20x^2 + 31x - 15 \;\big|\; x-1 \\ -4x^3 + 4x^2 \qquad\qquad 4x^2 - 16x + 15 \\ \hline -16x^2 + 31x - 15 \\ +16x^2 - 16x \\ \hline +15x - 15 \\ -15x + 15 \\ \hline 0 \end{array}$$

$\dfrac{x+4}{4x^2 - 16x + 15} \rightarrow \dfrac{x+4}{(x-\frac{3}{2})(x-\frac{5}{2})}$

$x^2 - 16x + 60 \begin{cases} x = 6 \rightarrow \frac{6}{4} = \frac{3}{2} \\ x = 10 \rightarrow \frac{10}{4} = \frac{5}{2} \end{cases}$

$D_f = \mathbb{R} - \{\frac{3}{2}, \frac{5}{2}\}$

ب) $y = \dfrac{x+3}{3x^3 - x^2 - 8x - 4}$

$x = -1$ ، $\frac{x+1}{?}$

$$\begin{array}{l} 3x^3 - x^2 - 8x - 4 \;\big|\; x+1 \\ -3x^3 - 3x^2 \qquad\qquad 3x^2 - 4x - 4 \\ \hline -4x^2 - 8x - 4 \\ +4x^2 + 4x \\ \hline -4x - 4 \\ +4x + 4 \\ \hline 0 \end{array}$$

$\dfrac{x+3}{3x^2 - 4x - 4} =$

$x^2 - 4x - 12$

$(x-6)(x+2) \rightarrow \begin{cases} x = \frac{6}{3} = 2 \\ x = \frac{2}{3} \end{cases}$

$\mathbb{R} - \{2, \frac{2}{3}\}$

2-

الف) $y = \dfrac{x+1}{x - \sqrt{3-2x}} \rightarrow \mathbb{R}$

$x - \sqrt{3-2x} = 0$ ، $3 - 2x \geq 0 \rightarrow 3 \geq 2x \rightarrow \frac{3}{2} \geq x$

$x = \sqrt{3-2x} \rightarrow x^2 = 3 - 2x$

$x^2 + 2x - 3 = 0 \rightarrow (x+3)(x-1)$ ، $x = \begin{cases} -3 \rightarrow \text{غ ق ق} \; \times \\ 1 \rightarrow \checkmark \end{cases}$

$(-\infty, \frac{3}{2}] \cap (\mathbb{R} - \{1\}) \rightarrow$

ب) $y = \dfrac{x+2}{x - \sqrt{3x-2}} \Rightarrow$

$3x - 2 \geq 0 \rightarrow x \geq \frac{2}{3}$

$x = \sqrt{3x-2} \rightarrow$ توان ۲

$x^2 = 3x - 2 \rightarrow$

$x^2 - 3x + 2 = 0 \rightarrow$

$(x-2)(x-1) \rightarrow \begin{cases} x = 2 \rightarrow \checkmark \\ x = 1 \rightarrow \checkmark \end{cases}$

$(\mathbb{R} - \{2, 1\}) \cap [+\frac{2}{3}, +\infty)$

3-

الف) $y = \dfrac{\sin x}{2\cos x - 1} \rightarrow \mathbb{R}$

$2\cos x - 1 \neq 0 \rightarrow \cos x \neq \frac{1}{2} \rightarrow$ ربع اول و چهارم

$\mathbb{R} - \{2k\pi + \frac{\pi}{3}, 2k\pi - \frac{\pi}{3}\}$

ب) $y = \dfrac{\cos x + 1}{2\sin x + 1} \rightarrow \mathbb{R}$

$\sin x \neq \frac{-1}{2} \rightarrow$ ربع سوم و چهارم

$\mathbb{R} - \{2k\pi - \frac{\pi}{6}, 2k\pi + \pi + \frac{\pi}{6}\}$

ج) $y = \frac{2\tan x + 1}{\cot x - 1}$ → $\tan x = \frac{\sin x}{\cos x}$

$\{k\pi + \frac{\pi}{2}\}$

$\begin{cases} \cos x \neq 0 \\ \cot x \neq 1 \rightarrow \frac{\cos x}{\sin x} \neq 1 \end{cases}$

$\mathbb{R} - \{k\pi + \frac{\pi}{2}, k\pi + \frac{\pi}{4}\}$ $\{k\pi + \frac{\pi}{4}\}$

د) $y = \frac{\sin x + 1}{4\sin^2 x - 3}$ → $\mathbb{R}$

$4\sin^2 x - 3 \neq 0$

$\sin^2 x \neq \frac{3}{4}$

$\left(\sin x \neq \frac{\sqrt{3}}{2}\right)$ → $60°$

$\mathbb{R} - \{2k\pi + \frac{\pi}{3}, 2k\pi + \pi - \frac{\pi}{3}\}$

-4

الف) $x^2 - 5x + 4 \geq 0$

$(x-4)(x+1) \rightarrow \begin{cases} x = 4 \\ x = -1 \end{cases}$

$-\infty \quad -1 \quad 4 \quad +\infty$

$+ \quad \phi \quad - \quad \phi \quad +$

$(4, +\infty) \cup (-\infty, -1)$

ب) $x^2 - 4x + 5 < 0$

$(x-5)(x-1) \rightarrow \begin{cases} x = 5 \\ x = 1 \end{cases}$

$1 \quad 5$

$+ \quad - \quad +$

$(1, 5)$

ج) $x^2 + 4x + 5 \geq 0$

$(x+5)(x+1) \rightarrow \begin{cases} x = -5 \\ x = -1 \end{cases}$

$-5 \quad -1$

$+ \quad - \quad + \rightarrow$

$[-1, +\infty) \cup (-\infty, -5]$

د) $x^2 - 2x + 4 \leq 0$

$(x-4)(x-1) \rightarrow \begin{cases} x = 4 \\ x = 1 \end{cases}$

$1 \quad 4$

$+ \quad - \quad + \rightarrow [1, 4]$

-5

الف) $\frac{x^2 - 3x + 2}{x - 5} < 0$

$x - 5 \rightarrow x = 5$

$(x-2)(x-1) \rightarrow \begin{cases} x = 2 \\ x = 1 \end{cases}$

$1 \quad 2 \quad 5$

$- \quad \phi \quad + \quad \phi \quad - \quad \| \quad +$

$(-\infty, 1) \cup (2, 5)$

ب) $\frac{x^2 - 1}{x^2 - 4x + 5} \geq 0$ (+1)

$x^2 - 4x + 5 \rightarrow (x-5)(x-1) \rightarrow x = \begin{cases} 5 \\ 1 \end{cases}$

$-1 \quad 1 \quad 5$

$+ \quad \phi \quad + \quad \phi \quad -$

$(5, +\infty) \cup (-1, 1)$

6-

الف) $y=\sqrt{\frac{x^2-4x+3}{x^3-1}}$

$x^3-1\neq 0 \rightarrow x^3\neq 1$

$x\neq 1$

$\frac{(x-3)(x-1)}{x^3-1}\geq 0$

$(-\infty,1)\cup[3,+\infty)$

ب) $y=\sqrt[3]{\frac{x^2+4x+3}{x^2-1}}$

$x^2-1\neq 0 \rightarrow x\neq \pm 1$

$\mathbb{R}-\{+1,-1\}$

7- الف) $y=\log(x^2-3x)$ $\quad (-\infty,0]\cup[3,+\infty)$

$x^2-3x>0 \rightarrow x(x-3)>0 \rightarrow x=0,3$

ب) $y=\log\frac{4-x^2}{|x|-2}$

$\begin{cases} 4-x^2>0 \rightarrow 4>x^2 \rightarrow -2<x<2 \\ |x|-2>0 \rightarrow |x|>2 \rightarrow x>2 \text{ یا } x<-2 \\ |x|-2\neq 1 \rightarrow |x|\neq 3 \rightarrow x\neq \pm 3 \end{cases}$

$D_f=(-2,2)-\{\pm 3\}$

ج) $y=\log\frac{x^2-3x}{x-3}$

$10-x$

$\begin{cases} \frac{x(x-3)}{x-3}>0 \rightarrow x-3>0 \rightarrow x>3 \\ 10-x\neq 1 \rightarrow x\neq 9 \\ 10-x>0 \rightarrow 10>x \end{cases}$

د) $y=\sqrt{\frac{x^2-11x+18}{x-4}}+\log\sqrt{9-x^2}$

$D_f=(3,10)-\{9\}$

$(x-2)(x-9)$

$x=2,9$

$+\ |\ -\ |\ +$

$(-\infty,2]\cup[9,+\infty)$

$\sqrt{9-x^2}>0 \rightarrow 9-x^2>0 \rightarrow -3<x<3$

$x>0$

$x\neq 1$

$(0,3)-\{1\}$

$(-\infty,2]\cup[9,+\infty)\cap(0,3)-\{1\}$

8- $y=\frac{x^2-x}{x^2+x}$

$x^2-x \rightarrow x(x-1) \rightarrow 0,1$

$x^2+x \rightarrow x(x+1) \rightarrow 0,-1$

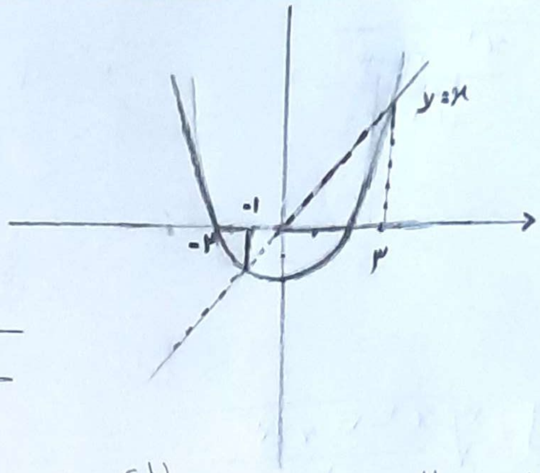
	$(-\infty,-1)$	-1	$(-1,0)$	0	$(0,1)$	1	$(1,+\infty)$
x^2-x	+	+	+	0	−	0	+
x^2+x	+	0	−	0	+	+	+
$P(x)$	+	∄	−	0	−	0	+

$D_f=(-\infty,-1)\cup(-1,0)\cup(1,+\infty)$

$R_f=(-\infty,0)\cup[1,+\infty)$

- 9

$y=\sqrt{\dfrac{x-f(x)}{f(x)}}$

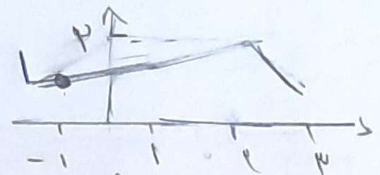


	-2		-1		2		3	
x−f(x)	+	|	+	0	−	|	−	0	+
f(x)	−	0	+	|	+	0	−	|	+
	−	∄	+	|	−	∄	+	|	+

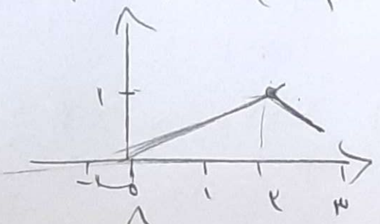
$[-1,-2) \cup (2,+\infty)$

- 10

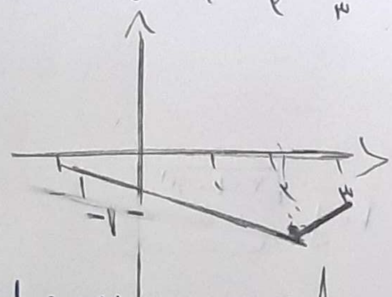
الف) f(x) + 1



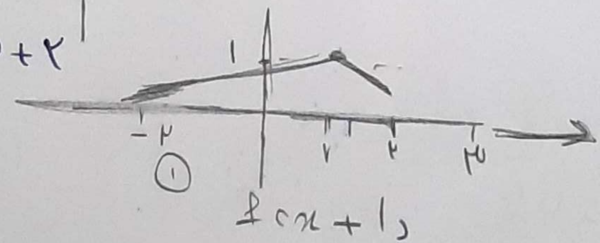
ب) f(x − 1)



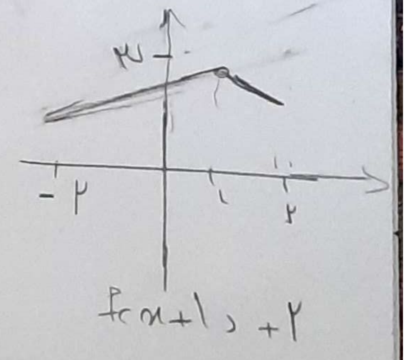
ج) − f(x)



د) f(x + 1) + 2



① f(x + 1)



② f(x + 1) + 2