

الف)  $y = x^2 - a \rightarrow x^2 = y + a \quad x = \pm \sqrt{y+a} \quad R_f = [-a, +\infty)$

(1) 5

ب)  $y = x^2 + 1 \rightarrow x^2 = y - 1 \quad x = \pm \sqrt{y-1} \quad R_f = R$

الف)  $y = x^2 - 2x + 4 \rightarrow y = x^2 - 2x + 1 - 1 + 4 \rightarrow y = (x-1)^2 - 1 + 4 \quad y = (x-1)^2 + 3 \rightarrow (x-1)^2 = y - 3$

(2) 5

$x-1 = \pm \sqrt{y-3} \rightarrow x = \sqrt{y-3} + 1$   
 $R_f = [3, +\infty)$   
 $y-3 \geq 0 \rightarrow y \geq 3$

ب)  $y = x^2 - ax + 1 \rightarrow y = x^2 - ax + \frac{a^2}{4} - \frac{a^2}{4} + 1 \rightarrow y = (x - \frac{a}{2})^2 - \frac{a^2}{4} + 1 \rightarrow y = (x - \frac{a}{2})^2 - \frac{a^2 - 4}{4} \rightarrow (x - \frac{a}{2})^2 = y + \frac{a^2 - 4}{4}$

$x - \frac{a}{2} = \pm \sqrt{y + \frac{a^2 - 4}{4}} \rightarrow x = \pm \sqrt{y + \frac{a^2 - 4}{4}} + \frac{a}{2}$   
 $R_f = [-\frac{a^2 - 4}{4}, +\infty)$   
 $y + \frac{a^2 - 4}{4} \geq 0 \rightarrow y \geq -\frac{a^2 - 4}{4}$

الف)  $y = \frac{x^2 + 3}{x^2 - 2} \rightarrow y(x^2 - 2) = x^2 + 3 \rightarrow yx^2 - 2y = x^2 + 3 \rightarrow yx^2 - x^2 = 2y + 3 \rightarrow x^2(y-1) = 2y+3 \rightarrow x^2 = \frac{2y+3}{y-1}$

(10) 5

$\frac{-\frac{3}{y-1}}{+ \quad - \quad +}$   
 $x = \pm \sqrt{\frac{2y+3}{y-1}}$   
 $R_f = (-\infty, -\frac{3}{y-1}] \cup (1, +\infty)$

ب)  $y = \frac{2|x|+1}{|x|-2}$

$y|x| - 2y = 2|x| + 1 \rightarrow y|x| - 2|x| = 2y + 1 \rightarrow |x|(y-2) = 2y+1 \rightarrow |x| = \frac{2y+1}{y-2}$

$\frac{-\frac{1}{y-2}}{+ \quad - \quad +}$   
 $R_f = (-\infty, -\frac{1}{y-2}] \cup (2, +\infty)$

$y = \frac{1}{x^2 - 12x}$

$\rightarrow yx^2 - 12yx = 1 \rightarrow yx^2 - 12yx - 1 = 0$

$x = \frac{-12y \pm \sqrt{\Delta}}{2y} \rightarrow \Delta \geq 0 \rightarrow |8y^2 + 12y| \geq 0$

(15) 5

$\frac{-\frac{1}{2y}}{+ \quad - \quad +}$

$8y(8y+12) \geq 0 \quad R_f = (-\infty, -\frac{1}{8}) \cup [0, +\infty)$

الف)  $y = x^2 - 4x + 2 \rightarrow a > 0 \rightarrow \begin{cases} x_{min} = \frac{-b}{2a} = 2 \\ y_{min} = -1 \end{cases}$

$R_f = [-1, +\infty)$

(2) 5

ب)  $y = -x^2 + 4x + 2 \rightarrow a < 0 \rightarrow \begin{cases} x_{max} = \frac{-b}{2a} = 2 \\ y_{max} = 6 \end{cases}$

$R_f = (-\infty, 6]$

الف)  $y = \sqrt{x^2 - 4x + 4} \xrightarrow{a > 0} \begin{cases} x_{\min} = \frac{-b}{2a} = 2 \\ y_{\min} = -V \end{cases}$

$R_f = \sqrt{[-V, +\infty)} = [0, +\infty)$

۲

۶

ب)  $y = \sqrt{-x^2 + 6x + 10} \xrightarrow{a < 0} \begin{cases} x_{\max} = \frac{-b}{2a} = 3 \\ y_{\max} = 14 \end{cases}$

$R_f = \sqrt{(-\infty, 14]} = [0, \sqrt{14}]$

الف)  $y = x^2 + 12x + 1 \quad R_f = \mathbb{R}$

۷

ب)  $y = \sqrt{x^2 + 6x + 10} \quad R_f = [0, +\infty)$

۲

الف)  $y = \frac{4x+1}{1x-2} \quad R_f = \mathbb{R} - \{2\}$

۸

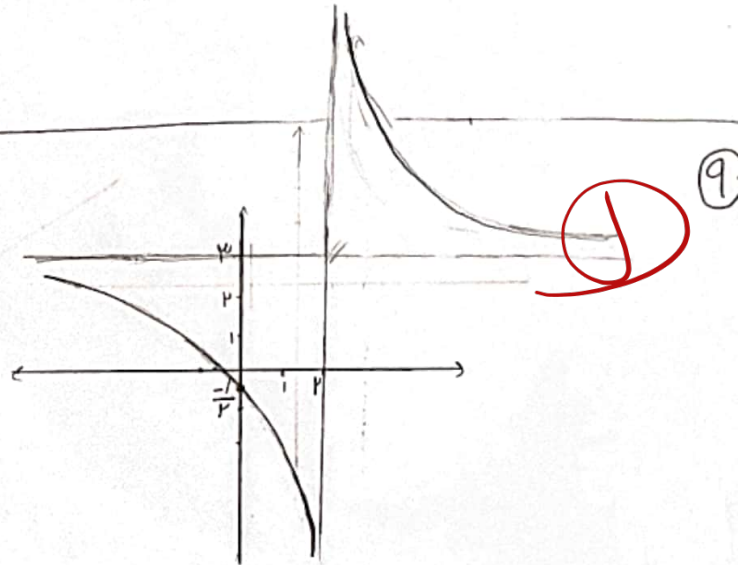
ب)  $y = \sqrt{\frac{4x+1}{x+3}} \quad R_f = [0, +\infty) - \{2\}$

$2 = x \rightarrow x^2 - 2x - 2 = 0$  - جانب قائم

الف)  $y = \frac{4x+1}{x-2} \rightarrow y = \frac{a}{c} = 2$  - جانب قائم

$\rightarrow 0$  - تقاطع

$4x+1=0 \rightarrow x = -\frac{1}{4}$



ب)  $y = \frac{4x-2}{1-2x} \rightarrow x = \frac{1}{2}$  - جانب قائم

$\rightarrow y = \frac{a}{c} = -2$  - جانب قائم

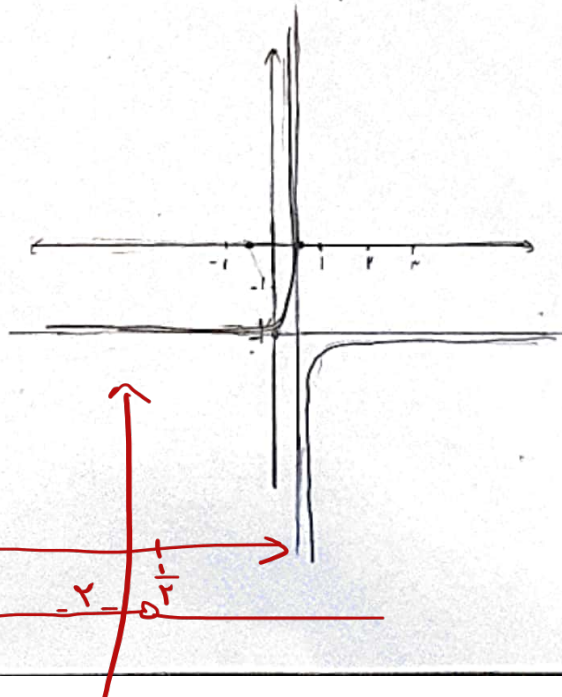
$\rightarrow -2$  - تقاطع

$4x-2=0 \rightarrow x = \frac{1}{2}$

همواره مثبت است

$\frac{2(2x-1)}{-(2x-1)} = -2$

$1-2x \neq 0 \rightarrow x \neq \frac{1}{2}$



الف)  $y = \cos^r x + \frac{1}{\cos^r x}$        $R_f = [r, +\infty)$

ر

ب)  $y = \sqrt[r]{\frac{x^r + 1}{x}}$

$R_f = (-\infty, \sqrt[r]{-r}] \cup [\sqrt[r]{r}, +\infty)$  ←  $\sqrt[r]{x + \frac{1}{x}}$

$\sqrt[r]{1}$