

1) الف) $y = a^2 - \Delta \Rightarrow a^2 = y + \Delta \quad a = \pm \sqrt{y + \Delta} \quad R_f = [-\infty, +\infty)$

ب) $a^2 = 1 \Rightarrow a^2 = y - 1 \quad a = \pm \sqrt{y - 1} \Rightarrow R_f = \mathbb{R}$

2) الف) $y = a^2 - \varepsilon a + \gamma \quad R_f = [\gamma + \infty)$

$y = a^2 - \varepsilon a + \gamma - \gamma + \gamma$

$y = (a - \frac{\varepsilon}{2})^2 + \gamma - \frac{\varepsilon^2}{4}$

$(a - \frac{\varepsilon}{2})^2 = y - \gamma + \frac{\varepsilon^2}{4}$

$a - \frac{\varepsilon}{2} = \pm \sqrt{y - \gamma + \frac{\varepsilon^2}{4}} \Rightarrow a = \pm \sqrt{y - \gamma + \frac{\varepsilon^2}{4}} + \frac{\varepsilon}{2}$

ب) $y = a^2 - \varepsilon a + 1$

$a^2 - \varepsilon a + 1 - y = 0$

$\Delta = \varepsilon^2 - 4(1 - y)$

$\Delta = \varepsilon^2 - 4 + 4y = 4y - 4 + \varepsilon^2$

$a = \frac{\varepsilon \pm \sqrt{4y - 4 + \varepsilon^2}}{2}$

$R_f = [\frac{-\varepsilon}{2}, +\infty)$

$4y - 4 + \varepsilon^2 \geq 0 \quad 4y \geq 4 - \varepsilon^2 \quad y \geq \frac{4 - \varepsilon^2}{4}$

3) الف) $y = \frac{a^2 + r}{a^2 - r} \Rightarrow y a^2 - r y = a^2 + r$

$a^2 - y a^2 = -r y - r \Rightarrow a^2(1 - y) = -r y - r$

$a^2 = \frac{-r y - r}{1 - y} \Rightarrow a^2 = \frac{r y + r}{y - 1}$

$a = \pm \sqrt{\frac{r y + r}{y - 1}}$

$R_f = (-\infty, \frac{-r}{r}] \cup (1, +\infty)$

$\frac{-r}{r} = -1$

$r y + r = 0$
 $r y = -r$
 $y = \frac{-r}{r}$

$y - 1 = 0$
 $y = 1$

ب) $y = \frac{r|a| + 1}{|a| - \varepsilon} \Rightarrow y|a| - \varepsilon y = r|a| + 1$

$r|a| - y|a| = -\varepsilon y - 1 \Rightarrow |a|(r - y) = -\varepsilon y - 1$

$|a| = \frac{-\varepsilon y - 1}{r - y}$

$R_f = (-\infty, \frac{-1}{\varepsilon}] \cup (\frac{1}{r}, +\infty)$

$r - y = 0 \quad \varepsilon y + 1 = 0$
 $y = r \quad \varepsilon y = -1$
 $y = \frac{-1}{\varepsilon}$

4) $y = \frac{1}{a^2 - \varepsilon a} \Rightarrow y a^2 - \varepsilon y a = 1 \Rightarrow y a^2 - \varepsilon y a - 1 = 0$

$a = \frac{\varepsilon y \pm \sqrt{\varepsilon^2 y^2 + 4y}}{2y}$

$\varepsilon^2 y^2 + 4y \geq 0 \Rightarrow \varepsilon^2 y^2 \geq -4y$
 $y \geq \frac{-4}{\varepsilon^2}$

$R_f = (-\infty, \frac{-1}{\varepsilon}] \cup [\frac{1}{\varepsilon}, +\infty)$

سوال ٥) الف) $y = ax^2 - 6ax + 2 \Rightarrow a > 0 \quad \begin{cases} x_{min} = \frac{-b}{2a} = \frac{+6}{2} = 3 \\ y_{min} = -V \end{cases} \quad R_f = [-V, +\infty)$

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ب) $y = -ax^2 + 6ax + 2 \quad a < 0 \quad \begin{cases} x_{max} = \frac{-b}{2a} = \frac{-6}{-2} = 3 \\ y_{max} = 6 \end{cases} \quad R_f = (-\infty, 6]$

سوال ٦) الف) $y = \sqrt{ax^2 - 6ax + 2} \quad a > 0 \quad \begin{cases} x_{min} = \frac{+6}{2} = 3 \\ y_{min} = -V \end{cases} \quad R_f = \sqrt{[-V, +\infty)} = R_f = [0, +\infty)$

ب) $y = \sqrt{-ax^2 + 6ax + 2} \quad a < 0 \quad \begin{cases} x_{max} = 3 \\ y_{max} = 12 \end{cases} \quad R_f = \sqrt{(-\infty, 12]} \Rightarrow R_f = [0, \sqrt{12}]$

سوال ٧) الف) $y = ax^2 + 2ax + 1 \rightarrow R_f = \mathbb{R}$

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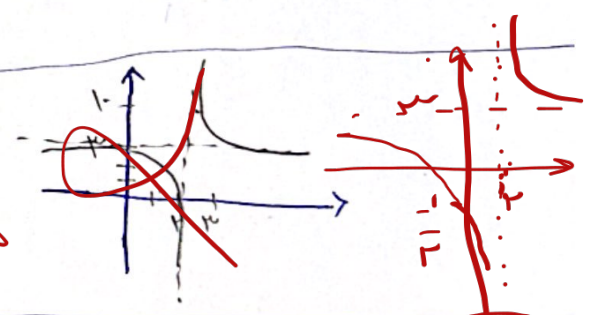
ب) $y = \sqrt{ax^2 + 2ax + 1} \rightarrow R_f = \sqrt{\mathbb{R}} = [0, +\infty)$

سوال ٨) الف) $y = \frac{2x+1}{x-2} \quad R_f = \mathbb{R} - \{2\}$
 $\frac{a}{c} = 2$

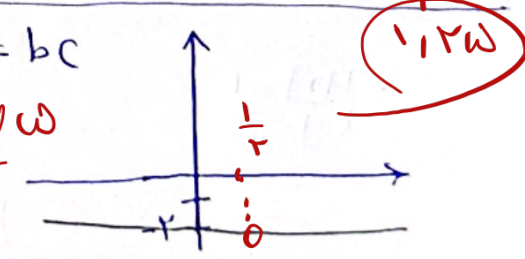
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ب) $y = \frac{2x+1}{\sqrt{x+3}} \quad R_f = [0, +\infty) - \{2\}$

سوال ٩) الف) $y = \frac{2x+1}{x-2}$
 $ax - 2 = 0 \Rightarrow x = 2$
 $n = 0, y = -\frac{1}{2}$
 $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$ نقطة كسرية



ب) $y = \frac{2x-2}{1-2x} \rightarrow \frac{2(x-1)}{1-2x}$
 $-2(-2) = 4 \quad ad \neq bc$
 $1-2x \neq 0 \Rightarrow x \neq \frac{1}{2}$



سوال ١٠) الف) $y = \cos^2 x + \frac{1}{\cos^2 x} \quad \cos^2 x > 0 \rightarrow R_f = [2, +\infty)$

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ب) $y = \sqrt{\frac{a^2+1}{a}} \Rightarrow \sqrt{a + \frac{1}{a}} \Rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$
 $a > 0 \rightarrow (1, +\infty)$
 $a < 0 \rightarrow (-\infty, -1]$