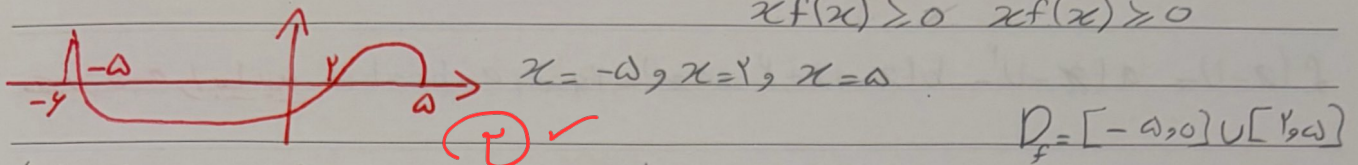


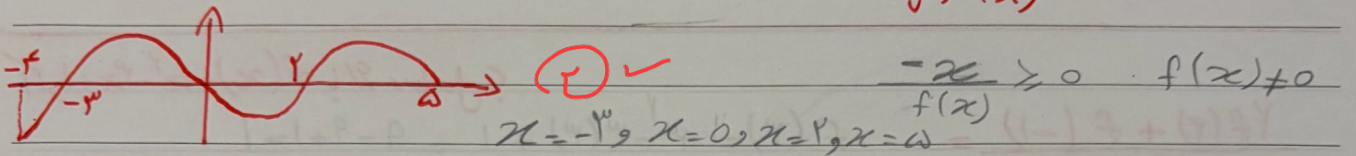
۱- دامنه ی تابع $y = \sqrt{x f(x)}$ را بدست آورید. x و $f(x)$ هم علامت باشند یا منفی.



منفی $\Rightarrow x < 0, f(x) > 0$: $(-5, -2)$: $x > 0, f(x) < 0$: $(2, 5)$

مثبت $\Rightarrow x < 0, f(x) < 0$: $(-2, 0)$: $x > 0, f(x) > 0$: $(0, 2)$

۲- دامنه ی تعریف $y = \sqrt{\frac{-x}{f(x)}}$ شامل چند عدد صحیح است؟ f



$f(x) \rightarrow (-4, -3)$: منفی $\rightarrow \frac{-x}{f(x)} = -$ و $+$

$f(x) \rightarrow (-3, 0)$: مثبت $\rightarrow \frac{-x}{f(x)} = +$ و $+$

$f(x) \rightarrow (0, 2)$: منفی $\rightarrow \frac{-x}{f(x)} = -$ و $-$

$f(x) \rightarrow (2, 5)$: مثبت $\rightarrow \frac{-x}{f(x)} = +$ و $-$

اگر $f(x) - 2f(2) = x^2 - 2x + 4$ باشد، مقدار $f(-2)$ را بدست آورید.

$x = 2$

$$f(2) - 2f(2) = 2^2 - 2(2) + 4 \rightarrow -f(2) = 4 - 4 + 4 = 4 \rightarrow f(2) = -4$$

$$f(-2) - 2f(2) = (-2)^2 - 2(-2) + 4 \rightarrow f(-2) - 2(-4) = 4 + 4 + 4 = 12$$

$$f(-2) + 8 = 12 \rightarrow f(-2) = 4$$

۳- در تابع $f(x) = \begin{cases} x - \sqrt{x+4} & x > 3 \\ 2x+3 & x \leq 3 \end{cases}$ مقدار $f(f(5)) + f(f(1))$ را بدست آورید.

$$f(5) = 5 - \sqrt{5+4} = 5 - 3 = 2 \quad f(f(5)) = f(2) \quad 5 > 3 \Rightarrow f(f(5)) = 2 - \sqrt{2+4} = 2 - \sqrt{6}$$

$$f(2) = 2(2) + 3 = 7 \quad f(f(5)) = 7$$

$$f(1) = 2(1) + 3 = 5 \quad f(f(1)) = f(5) = 2 - \sqrt{6}$$

$$f(f(5)) + f(f(1)) = 7 + 2 - \sqrt{6} = 9 - \sqrt{6}$$

تابع $f(x) = ax^2 - bx + 2$ را می دانیم $f(x-1) - f(x) = 4x + 2$ بیادای صحت دارد برقرار است.

$$f(x-1) = a(x-1)^2 - b(x-1) + 2 = ax^2 - 2ax + a - bx + b + 2$$

$$f(x-1) - f(x) = (ax^2 - 2ax + a - bx + b + 2) - (ax^2 - bx + 2)$$

$$= -2ax + a + b = 4x + 2 \rightarrow -2a = 4 \rightarrow a = -2$$

$$a + b = 2 \quad -2 + b = 2 \quad b = 4 \quad a - b = -2 - 4 = -6$$

(2) ✓

اگر $f(x) = x^2 - 3x + 1$ باشد، حاصل؟

$$2f(3) + f(-1) = f(3) = 3^2 - 3(3) + 1 = 9 - 9 + 1 = 1$$

$$f^2(2)$$

$$f(-1) = (-1)^2 - 3(-1) + 1 = 1 + 3 + 1 = 5$$

$$f^2(2) = f(f(2)) \rightarrow f(2) = 2^2 - 3(2) + 1 = 4 - 6 + 1 = -1$$

$$f^2(2) = f(-1) = 5 \quad \frac{2(1) + 5}{(-1)^2} = \frac{7}{1} = 7$$

اگر $f(x) = \begin{cases} 3x+1 & x \geq 2 \\ x^2 & x < 2 \end{cases}$ باشد، حاصل؟

$$\frac{3f(4) - 2f(-2)}{f(2)} = f(4) = 3(4) + 1 = 13 \quad 4 \geq 2$$

$$f(-2) = (-2)^2 = 4 \quad -2 < 2$$

$$f(2) = 3(2) + 1 = 7 \quad 2 \geq 2$$

$$\frac{3(13) - 2(4)}{7} = \frac{39 - 8}{7} = \frac{31}{7}$$

(3) ✓

اگر $f(2) = 6, f(3) = 2, f(4) = 1, f(1) = 4$ باشد، حاصل؟ $f = \{(2, 6), (3, 2), (4, 1), (1, 4)\}$

$$\frac{f(f(4)) - f(f(3))}{f(1) - 2} \quad f(4) = 1 \rightarrow f(f(4)) = f(1) = 4$$

$$f(3) = 2 \rightarrow f(f(3)) = f(2) = 6 \quad f(1) - 2 = 4 - 2 = 2$$

$$\frac{4 - 6}{2} = \frac{-2}{2} = -1$$

(4) ✓

اگر $f(x) = \frac{x^2 + 4x + 9}{x^2 + 4x + 5}$ با $f(x)$ مقادیر $f(f(\sqrt{3}-2))$ را بیابید.

$$f(\sqrt{3}-2) = \frac{(\sqrt{3}-2)^2 + 4(\sqrt{3}-2) + 9}{(\sqrt{3}-2)^2 + 4(\sqrt{3}-2) + 5} = \frac{3 - 4\sqrt{3} + 4 + 4\sqrt{3} - 8 + 9}{3 - 4\sqrt{3} + 4 + 4\sqrt{3} - 8 + 5} = 1$$

$$f(\sqrt{3}-2) = \frac{1}{1} = 1 \quad f(1) = \frac{1^2 + 4(1) + 9}{1^2 + 4(1) + 5} = \frac{14}{10} = \frac{7}{5}$$

✓

اگر $f(x) = x^2 + \frac{1}{x^2}$ با $f(x)$ مقادیر $(f(\sqrt{x}))^2 - f(x)$ را بیابید.

$$f(\sqrt{x}) = (\sqrt{x})^2 + \frac{1}{(\sqrt{x})^2} \rightarrow (\sqrt{x})^2 = x \quad f(\sqrt{x}) = x + \frac{1}{x}$$

$$(f(\sqrt{x}))^2 = \left(x + \frac{1}{x}\right)^2 = x^2 + 2(x)\left(\frac{1}{x}\right) + \frac{1}{x^2} = x^2 + 2 + \frac{1}{x^2}$$

$$\left(x^2 + 2 + \frac{1}{x^2}\right) - x^2 - \frac{1}{x^2} = 2$$

✓