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1, b

$x f(x) \geq 0$

$$\begin{cases} x = -4 \Rightarrow -4f(-4) < 0 \\ x = -5 \Rightarrow -5f(-5) = 0 \\ x = 2 \Rightarrow 2f(2) > 0 \\ x = 5 \Rightarrow 5f(5) > 0 \end{cases}$$

$[-5, 0] \cup [2, 5]$

1, b

$\frac{-x}{f(x)} \geq 0, f(x) \neq 0$

$$\begin{cases} x = 3 \Rightarrow \frac{3}{f(3)} = \frac{3}{0} \times \\ x = 2 \Rightarrow \frac{-2}{f(2)} = \frac{-2}{0} \times \end{cases}$$

$(-3, 0) \cup (0, 2) \Rightarrow -2, -1, 1 \Rightarrow 3$

$$\begin{aligned} f(x) - 2f(2) &= x^2 - 3x + 4 \\ f(2) - 2f(2) &= 2^2 - 3(2) + 4 \Rightarrow -f(2) = 4 - 6 + 4 \\ \Rightarrow -f(2) &= 2 \Rightarrow f(2) = -2 \end{aligned}$$

$$f(-2) - 2f(2) = (-2)^2 - 3(-2) + 4 \quad \text{✓}$$

$$f(-2) - 2(-2) = 4 + 6 + 4 \Rightarrow f(-2) + 4 = 14 \Rightarrow f(-2) = 10$$

$$f(x) = \begin{cases} x - \sqrt{x+4} & x \geq 3 \\ 2x + 3 & x \leq 3 \end{cases} \Rightarrow f(f(5)) + f(f(1)) \Rightarrow f(5) = 5 - \sqrt{5+4} = 5 - 3 = 2$$

$$f(f(5)) = f(2) \Rightarrow 2 \leq 3 \Rightarrow f(2) = 2(2) + 3 = 7$$

$$f(1) \Rightarrow 1 \leq 3 \Rightarrow f(1) = 2(1) + 3 = 5 \Rightarrow f(f(1)) = f(5) = 2$$

$$f(f(5)) + f(f(1)) = 7 + 2 = 9 \quad \text{✓}$$

$$f(x) = ax^2 - bx + c \Rightarrow f(x-1) - f(x) = 4x + 2 \Rightarrow f(x-1) = a(x-1)^2 - b(x-1) + c$$

$$a(x^2 - 2x + 1) - b(x-1) + c = ax^2 - 2ax + a - bx + b + c = ax^2 - (2a+b)x + (a+b+c)$$

$$f(x-1) - f(x) \Rightarrow ax^2 - (2a+b)x + (a+b+c) - (ax^2 - bx + c) = -2ax + a + b - bx + c = -2ax - bx + a + b + c = 4x + 2$$

$$-2a - b = 4 \Rightarrow a = -3 \Rightarrow a + b = 2, -3 + b = 2 \Rightarrow b = 5$$

$$a - b - c = -8 \Rightarrow -3 - 5 - c = -8 \Rightarrow c = 0$$

$\frac{r f(r) + f(-1)}{f'(r)} \Rightarrow f(r) = r^r - r \times r + 1 = 9 - 9 + 1 = 1$ $f(-1) = (-1)^r - r(-1) + 1 = 1 + r + 1 = 2$ $f(r) = r^r - r \times r + 1 = 9 - 9 + 1 = 1 \Rightarrow f'(r) = (-1)^r = 1$ $\Rightarrow \frac{r(1) + 2}{1} = \frac{r}{1} = \boxed{r}$	6
$\frac{r f(r) - r f(-r)}{f'(r)} \Rightarrow r \times 1 = r \times (-r) + 1 = -r^2 + 1$ $r^r = (-r)^r = r$ $r \times 1 = r \times (-r) + 1 = -r^2 + 1$ $\Rightarrow \frac{r(1) - r(-1)}{r} = \frac{r + 1}{r} = \boxed{\frac{r+1}{r}}$	7
$\frac{f(f(r)) - f(f(r))}{f'(r) - r} = \frac{9 - 9}{9 - r} = \frac{0}{9 - r} = \boxed{0}$	8
$\frac{r^r + r \times r + 9}{r^r + r \times r + 2} = \frac{(\sqrt{r} - r)^r + r(\sqrt{r} - r) + 9}{(\sqrt{r} - r) + r(\sqrt{r} - r) + 2}$ $\frac{r - r\sqrt{r} + r + r\sqrt{r} + 1 + 9}{r - r\sqrt{r} + r + r\sqrt{r} - 1 + 2} \Rightarrow r$ $\frac{1}{r} = r \Rightarrow f(r) = \frac{r^r + r(r) + 9}{r^r + r(r) + 2} = \frac{r + 1 + 9}{r + 1 + 2} = \boxed{\frac{r+10}{r+3}}$	9
$f(n) = n^r + \frac{1}{n^r} \Rightarrow f(\sqrt{n}) = (\sqrt{n})^r + \frac{1}{(\sqrt{n})^r} = n + \frac{1}{n}$ $\Rightarrow (f(\sqrt{n}))^r = (n + \frac{1}{n})^r \Rightarrow n^r + r + \frac{1}{n^r}$ $\Rightarrow f(n) = n^r + r + \frac{1}{n^r} \Rightarrow (f(\sqrt{n}))^r - f(n) = n^r + r + \frac{1}{n^r} - (n^r + \frac{1}{n^r}) = r$ $\Rightarrow \boxed{r}$	10