تکلیف ۲۵٪

★ ۱۹,۲۵ آذرین (ث)

سوال ۱) (۱,۷۵)

$$(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ) = \left(\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right)$$

$$\Rightarrow \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right) = \left(\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right) = \frac{2}{4} - \frac{3}{4} = \frac{-1}{4}$$ ✓

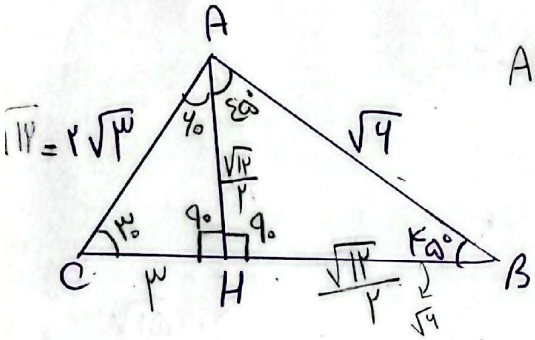
$$\frac{1 + 3\tan^2 120^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1 + 3\left(\frac{1}{\sqrt{3}}\right)^2}{4\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{-\sqrt{2}}{2}\right)^2} = \frac{1 + 3 \times \frac{1}{3}}{4 \times \frac{3}{4} + 2 \times \left(\frac{-2}{4}\right)}$$

توان ۲ مثبت است

$$\Rightarrow \frac{1+1}{3+(+1)} = \frac{2}{4} = \frac{1}{2}$$ ✓

سوال ۲)

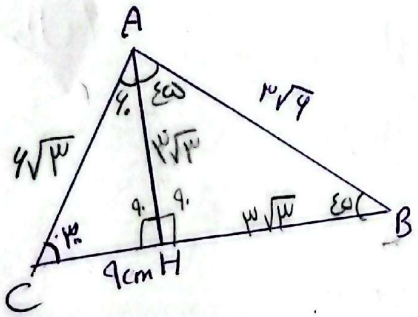
الف)



$$AH = BH \Rightarrow \frac{\sqrt{2}}{2} \times \sqrt{9} = \frac{\sqrt{12}}{2}$$

$$CH = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = \sqrt{9} \rightarrow CH = 3$$ ✓

ب)

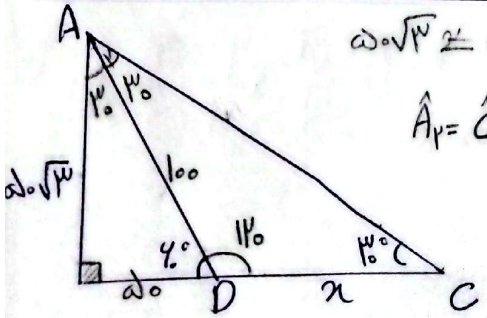


$$(3\sqrt{9})^2 - (3\sqrt{3})^2 = (9 \times 9) - (9 \times 3) = 27$$

$$HB^2 = 27 \rightarrow HB = \sqrt{27} = 3\sqrt{3}$$

$$AH = BH \Longrightarrow \hat{B} = \hat{A} = 45^\circ$$ ✓

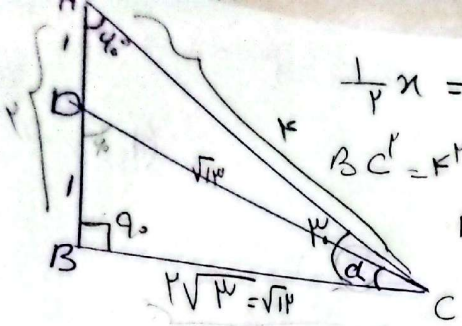
سوال ۳) الف)



$50\sqrt{3} = \frac{\sqrt{3}}{2} \times$ وتر $\rightarrow$ وتر $= 50\sqrt{3} \times \frac{2}{\sqrt{3}} = 100$

$\hat{A}_1 = \hat{C} = 30^\circ \rightarrow AD = DC = x$

$x = 100$ ✓



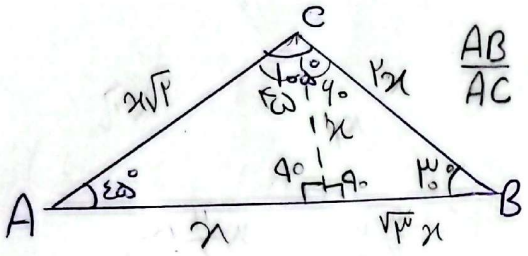
$\frac{۱}{۲}x = ۲ \rightarrow x = ۴ = AC$

$BC^۲ = ۴^۲ - ۲^۲ = ۱۶ - ۴ = ۱۲ \rightarrow BC = \sqrt{۱۲} = ۲\sqrt{۳}$

$DC = \sqrt{۱۳}$

$\sin\alpha = \frac{BD}{DC} = \frac{۱}{\sqrt{۱۳}}$ ✓

سوال ۷

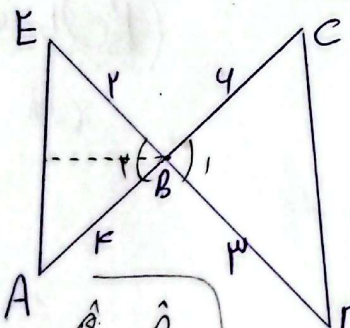


$\frac{AB}{AC} = \frac{x+\sqrt{۳}x}{x\sqrt{۲}} = \frac{x(۱+\sqrt{۳})}{x\sqrt{۲}} = \frac{۱+\sqrt{۳}}{\sqrt{۲}}$ ✓

$\frac{۱+\sqrt{۳}}{\sqrt{۲}} \times \frac{\sqrt{۲}}{\sqrt{۲}} = \frac{\sqrt{۲}+\sqrt{۶}}{۲}$

$x\sqrt{۲} \times \frac{\sqrt{۲}}{۲} =$

سوال ۵

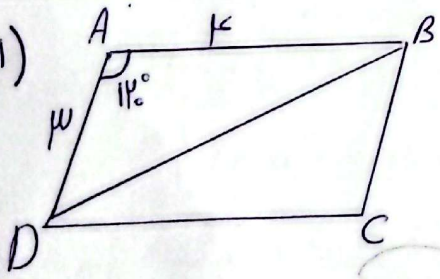


$\frac{S_{\triangle ABE}}{S_{\triangle BCD}} = \frac{\frac{۱}{۲} \times ۲ \times ۴ \times \sin\beta_۱}{\frac{۱}{۲} \times ۳ \times ۶ \times \sin\beta_۲} = \frac{۴}{۹}$ ✓

$\hat{B}_۱ = \hat{B}_۲$ دلیل: زاویه‌های متقابل به رأس

سوال ۹

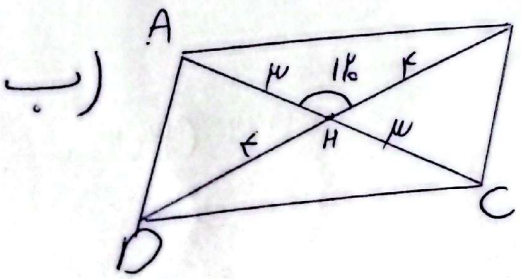
الف)



$S = AB \times AD \times \sin ۱۲۰°$

$S = ۴ \times ۳ \times \frac{\sqrt{۳}}{۲} = ۶\sqrt{۳}$ ✓

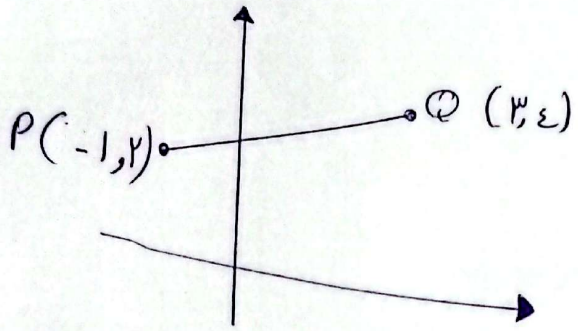
ب)



$S = \frac{۱}{۲} AH \times BH \times \sin ۱۲۰°$

$S = ۴ \times ۳ \times \frac{۱}{۳} \sin ۱۲۰° = ۴ \times ۳ \times \frac{\sqrt{۳}}{۲} \times \frac{۱}{۲} = ۳\sqrt{۳}$ ✓

سوال ۷



$ax+b=y \quad ax'+b=y'$
$-a+b=۲ \quad ۳a+b=٤$

۴۵

$\begin{cases} -a+b=۲ \\ (۳a+b=٤)\times -۱ \end{cases}$

$-a+b=۲$
$-۳a-b=-٤$
$-۴a=-۲ \rightarrow a=\frac{۱}{۲}$

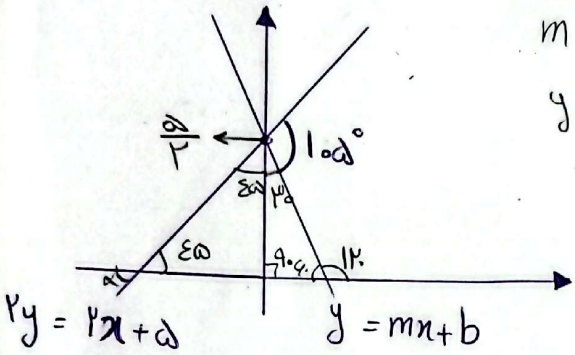
$-\frac{۱}{۲}+b=۲$
$b=۲+\frac{۱}{۲}=\frac{۵}{۲}$

$\frac{۱}{۲}x+\frac{۵}{۲}=y$

$a=\tan\theta=\frac{۱}{۲} \rightarrow \sin\theta=۱$
$\cos A=۲$ ✓ $-۱\le \cos\theta \le ۱$ ☹

$\tan\theta=\frac{۱}{۲} \rightarrow ۱+\tan^۲\theta=\frac{۱}{\cos^۲\theta}$
$\frac{۱}{\cos^۲\theta}=\frac{۵}{۴} \rightarrow \cos^۲\theta=\frac{۴}{۵}=\frac{۲}{\sqrt{۵}}$

سوال ۸



$m=\tan ۱۲۰^\circ=-\tan ۶۰=-\sqrt{۳} \Rightarrow a=-\sqrt{۳}$

$y=ax+b \Rightarrow mb=\frac{۵}{۲}(-\sqrt{۳}) \Rightarrow mb=\frac{-۵\sqrt{۳}}{۲}$ ✓

$y=x+۲/۵$
$\tan\alpha=۱ \Rightarrow a=۱$

سوال ۹

$f(x)=\underbrace{\cos(\pi+x)}_{-\cos x}+\underbrace{\sin(\frac{\pi}{۲}-x)}_{\cos x}-\underbrace{\tan(\frac{۳\pi}{۲}+x)}_{\frac{۳\pi}{۲}+x=\pi+\frac{\pi}{۲}+x=\tan(\frac{\pi}{۲}+x)=-\cot x}$

$f(x)=-\cos x+\cos x-(-\cot x) \Rightarrow f(x)=-\cos x+\cos x+\cot x$

$\Rightarrow f(x)=\cot x$
$x=\frac{۵\pi}{۶}=۱۵۰^\circ$
$\Rightarrow \cot(\frac{۵\pi}{۶})=\cot(\pi-\frac{\pi}{۶})=-\cot(\frac{\pi}{۶})=-\sqrt{۳}$ ✓

سوال ۱۰

$\frac{۱۷\pi}{۲}=۸\pi+\frac{\pi}{۲} \rightarrow \sin(۸\pi+\frac{\pi}{۲}+x) \Rightarrow \sin(\frac{\pi}{۲}+x)=\cos x$ ①

$\sin(\frac{۱۷\pi}{۲}+x)=\cos x$

$\frac{۱۵\pi}{۲}=۷\pi+\frac{\pi}{۲} \rightarrow \cos(۷\pi+\frac{\pi}{۲}-x) \Rightarrow \cos(\frac{-\pi}{۲}-x)=-\sin x$ ②

$\cos x+(-\sin x)=\cos x-\sin x$

$\Rightarrow \frac{\cos x-\sin x}{-\cos x+\sin x}$

③ $\cos(۳\pi+x)=\cos(\pi+x)=-\cos x$

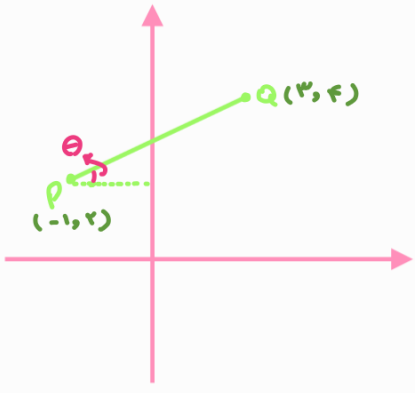
④ $\sin(۴\pi-x)=-\sin x$

$\hookrightarrow \frac{-(-\cos x+\sin x)}{-\cos x+\sin x}=-۱$

الف) $(\sin 135^\circ + \sin 30^\circ)(\cos 210^\circ - \cos 150^\circ)$

$$\left(\frac{\sqrt{2}}{2} - \sqrt{\frac{3}{2}}\right)\left(\frac{\sqrt{2}}{2} - \left(-\sqrt{\frac{3}{2}}\right)\right) = \left(\frac{\sqrt{2}}{2}\right)^2 - \left(\sqrt{\frac{3}{2}}\right)^2 = \frac{1}{2} - \frac{3}{4} = \boxed{-\frac{1}{4}}$$

ب) $$\frac{1 + \tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1 + 3\left(\frac{\sqrt{3}}{2}\right)^2}{4\left(\sqrt{\frac{3}{2}}\right)^2 + 2\left(-\frac{\sqrt{2}}{2}\right)^2} = \frac{1+1}{3+1} = \boxed{\frac{1}{2}}$$



$$m_{PQ} = \frac{4-2}{3-(-1)} = \frac{1}{2} = \tan\theta \rightarrow 1 + \tan^2\theta = \frac{1}{\cos^2\theta}$$

$$1 + \frac{1}{4} = \frac{1}{\cos^2\theta} \rightarrow \cos^2\theta = \frac{4}{5} \xrightarrow[\cos\theta > 0]{\text{ناحیه } \theta} \cos\theta = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$