تکلیف شماره ۲۰

آواحاتمی - دهم تجربی

سوال ۱)

$$(\sin 135^\circ + \sin 300^\circ)(\cos 315^\circ - \cos 150^\circ) = \left(\frac{\sqrt{2}}{2} + \left(-\frac{\sqrt{3}}{2}\right)\right)\left(\frac{\sqrt{2}}{2} - \left(-\frac{\sqrt{3}}{2}\right)\right)$$

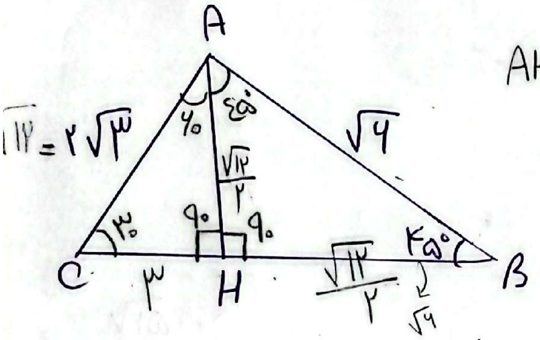
$$\Rightarrow \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2}\right) = \left(\left(\frac{\sqrt{2}}{2}\right)^2 - \left(\frac{\sqrt{3}}{2}\right)^2\right) = \frac{2}{4} - \frac{3}{4} = \boxed{-\frac{1}{4}}$$

$$\frac{1 + 3\tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1 + 3\left(\frac{1}{\sqrt{3}}\right)^2}{4\left(\frac{\sqrt{3}}{2}\right)^2 + 2\left(\frac{-\sqrt{2}}{2}\right)^2} = \frac{1 + 3 \times \frac{1}{3}}{4 \times \frac{3}{4} + 2 \times \left(\frac{-2}{4}\right)}$$

$$\Rightarrow \frac{1+1}{3+(-1)} = \frac{2}{2} = \boxed{1}$$

سوال ۲)

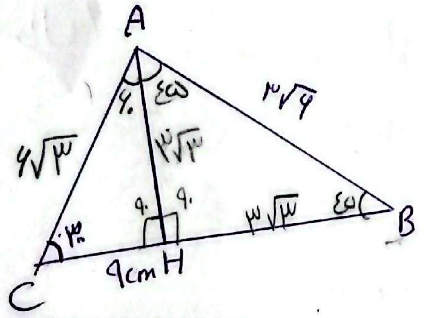
الف)



$$AH = BH \Rightarrow \frac{\sqrt{2}}{2} \times \sqrt{6} = \frac{\sqrt{12}}{2}$$

$$CH = \frac{\sqrt{3}}{2} \times 2\sqrt{3} = \sqrt{9} \rightarrow \boxed{CH = 3}$$

ب)

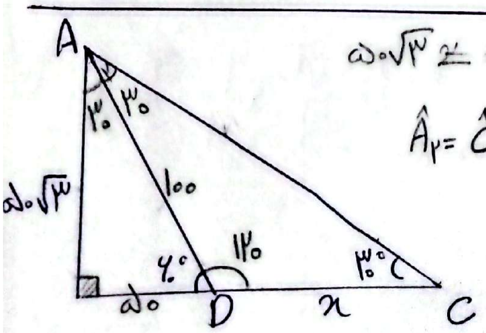


$$(3\sqrt{6})^2 - (3\sqrt{3})^2 = (9 \times 6) - (9 \times 3) = 27$$

$$HB^2 = 27 \rightarrow HB = \sqrt{27} = 3\sqrt{3}$$

$$AH = BH \Longrightarrow \hat{B} = \hat{A} = \boxed{45^\circ}$$

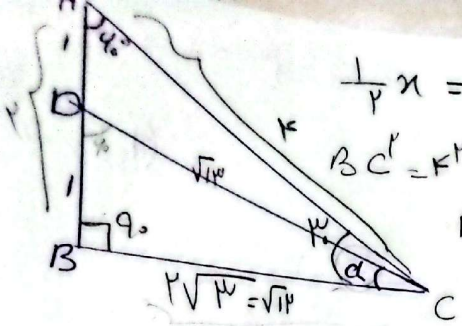
سوال ۳) الف)



$$50\sqrt{3} \cong \frac{\sqrt{3}}{2} \times \text{وتر} \rightarrow 50\sqrt{3} \times \frac{2}{\sqrt{3}} = 100 \ \text{وتر}$$

$$\hat{A}_2 = \hat{C} = 30^\circ \rightarrow AD = DC = x$$

$$\boxed{x = 100}$$



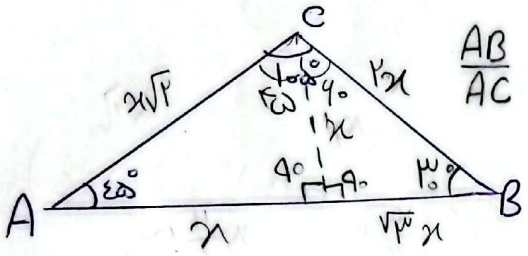
$\frac{1}{2}x = 2 \rightarrow x = 4 = AC$

$BC^2 = 6^2 - 2^2 = 16 - 4 = 12 \rightarrow BC = \sqrt{12} = 2\sqrt{3}$

$DC = \sqrt{13}$

$\sin\alpha = \frac{BD}{DC} = \frac{1}{\sqrt{13}}$ ✓

سوال ۷

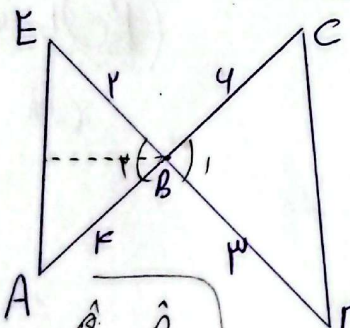


$\frac{AB}{AC} = \frac{x+\sqrt{3}x}{x\sqrt{2}} = \frac{x(1+\sqrt{3})}{x\sqrt{2}} = \frac{1+\sqrt{3}}{\sqrt{2}}$ ✓

$\frac{1+\sqrt{3}}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}+\sqrt{6}}{2}$

$x\sqrt{2} \times \frac{\sqrt{2}}{2} =$

سوال ۵

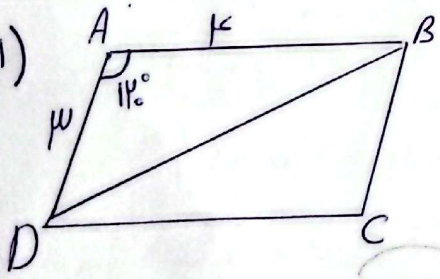


$\frac{S_{\triangle ABE}}{S_{\triangle BCD}} = \frac{\frac{1}{2} \times 2 \times 4 \times \sin\hat{B}_1}{\frac{1}{2} \times 3 \times 6 \times \sin\hat{B}_2} = \frac{4}{9}$ ✓

$\hat{B}_1 = \hat{B}_2$ دو زاویه متقابل به رأس

سوال ۹

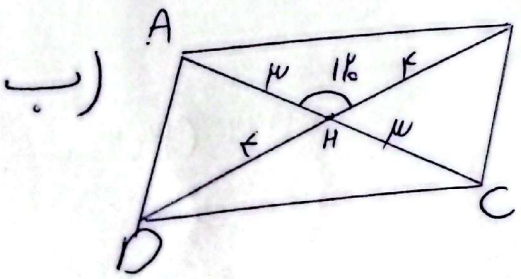
الف)



$S = AB \times AD \times \sin 120°$

$S = 4 \times 3 \times \frac{\sqrt{3}}{2} = 6\sqrt{3}$ ✓

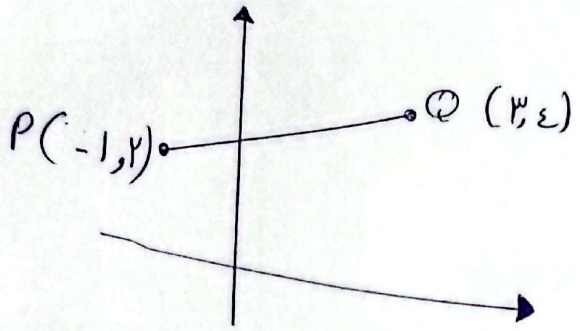
ب)



$S = \frac{1}{2} AH \times BH \times \sin 120°$

$S = 4 \times 3 \times \frac{1}{3} \sin 120° = \frac{12}{2} \times 3 \times \frac{\sqrt{3}}{2} \times \frac{1}{2} = 3\sqrt{3}$ ✓

سوال ۷



$$\begin{cases} ax+b=y \\ -a+b=۲ \end{cases} \quad \begin{cases} ax'+b=y' \\ ۳a+b=٤ \end{cases}$$

$$\begin{cases} -a+b=۲ \\ (۳a+b=٤)\times -۱ \end{cases} \qquad \begin{array}{r} -a+b=۲ \\ -۳a-b=-٤ \\ \hline -٤a=-۲ \end{array} \Rightarrow a=\frac{۱}{۲}$$

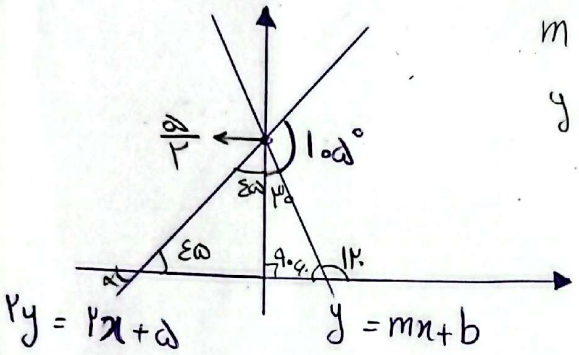
$$\frac{-۱}{۲}+b=۲$$

$$b=۲+\frac{۱}{۲}=\frac{۵}{۲}$$

$$\frac{۱}{۲}x+\frac{۵}{۲}=y$$

$$a=\tan\theta=\frac{۱}{۲} \rightarrow \begin{array}{l}\sin\theta=۱ \\ \cos A=۲\end{array}$$ ✓

سوال ۸



$$m=\tan ۱۲۰^\circ=-\tan ۶۰=-\sqrt{۳} \Rightarrow a=-\sqrt{۳}$$

$$y=ax+b \Rightarrow mb=\frac{۵}{۲}(-\sqrt{۳}) \Rightarrow mb=\frac{-۵\sqrt{۳}}{۲}$$ ✓

$$y=x+۲/۵$$

$$\tan\alpha=۱ \Rightarrow a=۱$$

سوال ۹

$$f(x)=\underbrace{\cos(\pi+x)}_{-\cos x}+\underbrace{\sin(\frac{\pi}{۲}-x)}_{\cos x}-\underbrace{\tan(\frac{۳\pi}{۲}+x)}_{\frac{۳\pi}{۲}+x=\pi+\frac{\pi}{۲}+x=\tan(\frac{\pi}{۲}+x)=-\cot x}$$

$$f(x)=-\cos x+\cos x-(-\cot x) \Rightarrow f(x)=-\cos x+\cos x+\cot x$$

$$\Rightarrow f(x)=\cot x$$

$$x=\frac{۵\pi}{۶}=۱۵۰^\circ$$

$$\Rightarrow \cot(\frac{۵\pi}{۶})=\cot(\pi-\frac{\pi}{۶})=-\cot(\frac{\pi}{۶})=-\sqrt{۳}$$ ✓

سوال ۱۰

$$\frac{۱۷\pi}{۲}=۸\pi+\frac{\pi}{۲} \rightarrow \sin(۸\pi+\frac{\pi}{۲}+x) \overset{①}{\Rightarrow} \sin(\frac{\pi}{۲}+x)=\cos x$$

$$\sin(\frac{۱۷\pi}{۲}+x)=\cos x$$

$$\frac{۱۵\pi}{۲}=۷\pi+\frac{\pi}{۲} \rightarrow \cos(۷\pi+\frac{\pi}{۲}-x) \overset{②}{\Rightarrow} \cos(\frac{-\pi}{۲}-x)=-\sin x$$

③ $\cos(۳\pi+x)=\cos(\pi+x)=-\cos x$

④ $\sin(۴\pi-x)=-\sin x$

$$\cos x+(-\sin x)=\cos x-\sin x$$

$$\Rightarrow \frac{\cos x-\sin x}{-\cos x+\sin x} \hookrightarrow \frac{-(-\cos x+\sin x)}{-\cos x+\sin x}=-۱$$