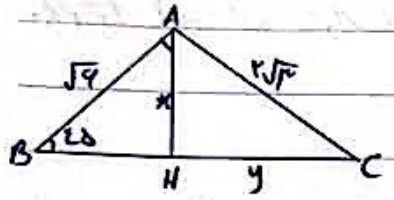
۱- الف) $(\sin 135 + \sin 30)(\cos 315 - \cos 150) = (\frac{\sqrt{2}}{2} + \frac{-\sqrt{3}}{2})(\frac{\sqrt{2}}{2} - \frac{-\sqrt{3}}{2}) = (\frac{\sqrt{2}-\sqrt{3}}{2})(\frac{\sqrt{2}+\sqrt{3}}{2}) =$

$\frac{2}{4} - \frac{3}{4} = \frac{-1}{4}$

ب) $\frac{1+3\tan^2 210}{4\cos^2 330 + 2\cos^2 225} = \frac{1+3(\frac{\sqrt{3}}{3})^2}{4(\frac{\sqrt{3}}{2})^2 + 2(\frac{-\sqrt{2}}{2})^2} = \frac{1+1}{3+1} = \frac{2}{4} = \frac{1}{2}$

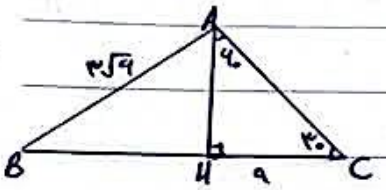
۲- الف)



$x\sqrt{2} = \sqrt{9} \Rightarrow x = \sqrt{3} = AH = BH$

$AH^2 + HC^2 = (2\sqrt{3})^2 = 12 \Rightarrow 3 + HC^2 = 12 \Rightarrow HC^2 = 9 \Rightarrow HC = 3$

ب)

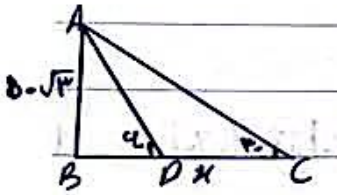


$\sin 90 = \frac{\sqrt{3}}{2} \quad \frac{9\times 2}{\sqrt{3}} = \frac{18}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = 6\sqrt{3} = AC$

$AH^2 + CH^2 = (6\sqrt{2})^2 = 108 \Rightarrow 81 + AH^2 = 108 \Rightarrow AH^2 = 27 \Rightarrow AH = 3\sqrt{3}$

$\sin B = \frac{AH}{AB} = \frac{3\sqrt{3}}{3\sqrt{6}} = \frac{\sqrt{3}}{\sqrt{6}} \times \frac{\sqrt{6}}{\sqrt{6}} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2} \Rightarrow B = 45^\circ$

۳- الف)

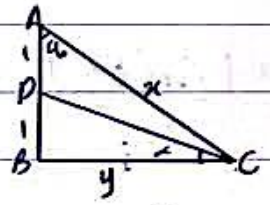


$\sin 30 = \frac{1}{2} \Rightarrow AC = 2\times 50\sqrt{3} = 100\sqrt{3} \quad \cos 30 = \frac{\sqrt{3}}{2} \Rightarrow BC = \frac{2\times 50\sqrt{3}}{\sqrt{3}} = 200$

$\sin 45 = \frac{\sqrt{2}}{2} \Rightarrow AD = \frac{2\times 50\sqrt{3}}{\sqrt{2}} = 100 \quad \cos 45 = \frac{1}{2} \Rightarrow \frac{1}{2}\times 100 = 50$

$CD = BC - BD = 200 - 50 = 150$

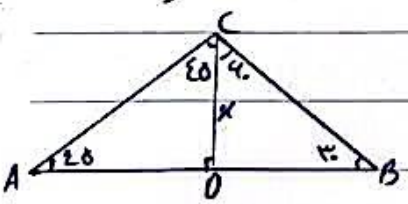
ب)



$\cos 45 = \frac{AB}{AC} = \frac{1}{2} = \frac{2}{x} \Rightarrow x = AC = 4 \quad \sin 45 = \frac{\sqrt{3}}{2} = \frac{BC}{AC} = \frac{y}{4} \Rightarrow BC = y = 2\sqrt{3}$

$DC^2 = BD^2 + BC^2 \Rightarrow DC = \sqrt{(\sqrt{3})^2 + 1} = \sqrt{13} \Rightarrow \sin\alpha = \frac{BD}{CD} = \frac{1}{\sqrt{13}} = \frac{\sqrt{13}}{13}$

۴-

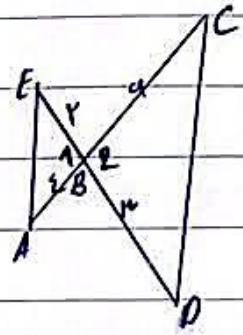


$CD = AD = x \Rightarrow AC = x\sqrt{2} \quad \sin 30 = \frac{1}{2} = \frac{CD}{BC} = \frac{x}{2x} \Rightarrow BC = 2x$

$\sin 45 = \frac{\sqrt{2}}{2} = \frac{BD}{BC} = \frac{x\sqrt{3}}{2x} \Rightarrow BD = x\sqrt{2} \Rightarrow \frac{AC}{AD+BD} = \frac{x\sqrt{2}}{x + x\sqrt{3}} = \frac{x\sqrt{2}}{x(1+\sqrt{3})}$

$\frac{\sqrt{2}}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{\sqrt{6}-\sqrt{2}}{2}$

۵-



$B_1 = B_2 \quad S_{EBA} = \frac{1}{2} \times EB \times BA \times \sin B_1 \quad S_{BCD} = \frac{1}{2} \times BC \times BD \times \sin B_2 = 5$

$\frac{S_{ABE}}{S_{BCD}} = \frac{EB \times AB}{BC \times BD} = \frac{2\times 4}{6\times 3} = \frac{8}{18} = \frac{4}{9}$

۹- الف) $S_{ABCD} = \frac{1}{2} \times 2 \times AB \times AD \times \sin\alpha = AB \times AD \times \sin\alpha \Rightarrow 3 \times 4 \times \sin 120 = 3 \times 4 \times \frac{\sqrt{3}}{2} = 9\sqrt{3}$

ب) از ضلع روش بالا برابست آورده : $4 \times \frac{1}{2} \times AB \times AC \times \sin\alpha \Rightarrow 4 \times 1,5 \times 2 \times \frac{1}{2} \times \sin 120 = 4 \times \frac{1}{2} \times 1,5 \times 2 \times \frac{\sqrt{3}}{2} = 3\sqrt{3}$

۷- $\cos\theta = \frac{\text{مجاور}}{\text{وتر}} = \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} \Rightarrow \frac{\Delta x}{\sqrt{\Delta x^2 + \Delta y^2}} = \frac{3-(-1)}{\sqrt{(3+1)^2 + (4-2)^2}} = \frac{4}{\sqrt{16+4}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

۸-

۹- $f\left(\frac{5\pi}{4}\right) = \cos\left(\pi + \frac{5\pi}{4}\right) + \sin\left(\frac{\pi}{2} - \frac{5\pi}{4}\right) - \tan\left(\frac{3\pi}{2} + \frac{5\pi}{4}\right) = \cos\left(\frac{5\pi}{4}\right) - \cos\left(\frac{5\pi}{4}\right) - \cot\left(\frac{5\pi}{4}\right)$

$-\cot\left(5\frac{\pi}{4}\right) = -\frac{\cos 150}{\sin 150} = \frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \sqrt{3}$

۶- $\frac{\cos(\pi) - \sin(\pi)}{-\cos(\pi) + \sin(\pi)} = \frac{-(\sin\pi - \cos\pi)}{\sin\pi - \cos\pi} = -1$

$\frac{\sin\left(\frac{\pi}{2} + \pi\right) + \cos\left(\frac{3\pi}{2} - \pi\right)}{\cos(\pi + \pi) - \sin(2\pi - \pi)}$