سایا داودی تکلیف ۲۰ گروه دهم دختر B

۱) الف) $(\sin 135^\circ + \sin 30^\circ)(\underbrace{\cos 315^\circ - \cos 150^\circ}_{\sin 135^\circ - \sin 30^\circ}) = \sin^2 135^\circ - \sin^2 30^\circ$

$(\frac{\sqrt{2}}{2})^2 - (-\frac{\sqrt{3}}{2})^2 = \frac{2}{4} - \frac{3}{4} = \boxed{-\frac{1}{4}}$

ب) $\frac{1+3\tan^2 210^\circ}{4\cos^2 330^\circ + 2\cos^2 225^\circ} = \frac{1+3(\frac{\sqrt{3}}{3})^2}{4(\frac{\sqrt{3}}{2})^2 + 2(-\frac{\sqrt{2}}{2})^2} = \frac{1+3(\frac{1}{3})}{4(\frac{3}{4})+2(\frac{2}{4})} = \frac{2}{4} = \boxed{\frac{1}{2}}$

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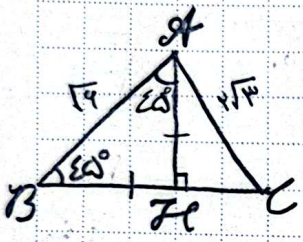
۲) الف)

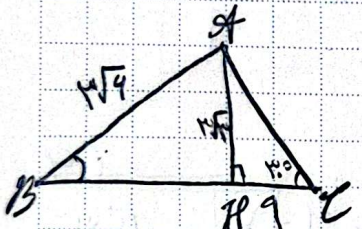


$\boxed{AH = \sqrt{3}}$

در مثلث متساوی الساقین وتر $a\sqrt{2}$ است و a طول ساق

$\rightarrow a\sqrt{2} = \sqrt{6} \rightarrow a = \sqrt{3}$

ب)



ضلع مجاور به 30° وتر است $\frac{\sqrt{3}}{2}$

$AH = \frac{\sqrt{3}}{2} AC$

$9 = \frac{\sqrt{3}}{2} AC \rightarrow AC = \frac{18}{\sqrt{3}} = 6\sqrt{3}$

ضلع مقابل به 30° وتر است $\frac{1}{2}$

$AH = \frac{1}{2} AC \rightarrow AH = \frac{1}{2} \times 6\sqrt{3} = 3\sqrt{3}$

★ روش آسان‌تر: بر اساس 30° ضلع مقابل $\tan 30^\circ$ ضلع مجاور است. $\frac{\sqrt{3}}{3}$

$\rightarrow \sin \hat{B} = \frac{3\sqrt{3}}{3\sqrt{6}} = \frac{\sqrt{2}}{2} \rightarrow \boxed{\hat{B} = 45^\circ}$

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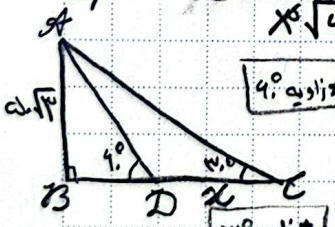
۳) الف)



★ زاویه 60°

$AB = \frac{\sqrt{3}}{2} AD \rightarrow AD = \frac{50\sqrt{3} \times 2}{\sqrt{3}} = 100$

★ زاویه 30°

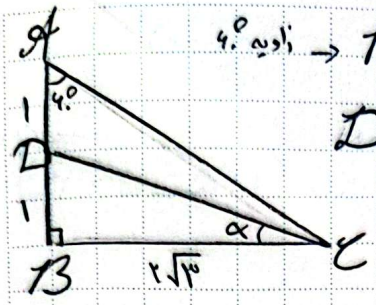
$AB = \frac{1}{2} AC \rightarrow AC = \frac{50\sqrt{3} \times 2}{1} = 100\sqrt{3}$

$BD = \frac{1}{2} AD \rightarrow BD = \frac{1 \times 100}{2} = 50$

$BC = \frac{\sqrt{3}}{2} AC \rightarrow BC = \frac{100\sqrt{3} \times \sqrt{3}}{2} = 150$

$BC - BD = x \rightarrow 150 - 50 = 100 \rightarrow \boxed{x = 100}$

۳) ب)



زاویه ۶۰° → $BC = \sqrt{3}\, AB \rightarrow BC = 2\sqrt{3}$

$DC = \sqrt{1+12} = \sqrt{13}$

$$\sin\alpha = \frac{1}{\sqrt{13}}$$

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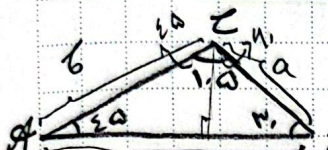
۴) ج)



$$\frac{AB}{AC} = \frac{c}{b} \Rightarrow \frac{c}{\frac{2c-\sqrt{2}a}{\sqrt{2}}} = \frac{\sqrt{2}c}{2c-\sqrt{2}a} = \frac{\sqrt{2}+\sqrt{4}}{2+2\sqrt{3}-2\sqrt{3}}$$

$$\frac{c}{b} = \frac{\sqrt{2}+\sqrt{4}}{2}$$

$$a\cos B + b\cos A = c \rightarrow b = \frac{c - a\cos(30^\circ)}{\cos(45^\circ)} = \frac{c - \frac{\sqrt{3}}{2}a}{\frac{\sqrt{2}}{2}} = \frac{2c-\sqrt{3}a}{\sqrt{2}}$$

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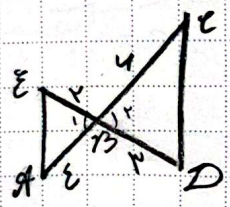
۵)



$$\frac{S_{ABE}}{S_{BED}} = \frac{\frac{1}{2}\times 2\times 4\times \sin\hat{B}_1}{\frac{1}{2}\times 3\times 4\times \sin\hat{B}_2} = \frac{4}{9}$$

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۶)

الف)
$$\frac{1}{2}\times 3\times 4\times \frac{\sqrt{3}}{2} = 4\sqrt{3}$$

ب)
$$\frac{1}{2}\times 3\times 4\times \frac{\sqrt{3}}{2} = 3\sqrt{3}$$

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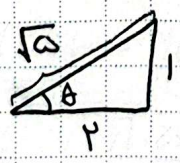
۷)

$y = ax + b \qquad b_1 = b_2$

$-2 = -1a + b_1$

$4 = 2a + b_2$

$\rightarrow 2 = 4a \rightarrow a = \frac{1}{2} \rightarrow \tan\theta = \frac{1}{2}$



$$\cos\theta = \frac{2}{\sqrt{5}}$$

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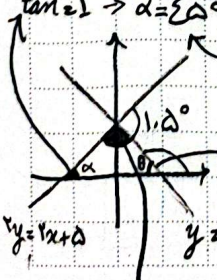
۸)



$y = 2x + a \Rightarrow y = x + \frac{a}{2}$

$b = \frac{a}{2}$ → محل قطع کردن محور y ها، یا عرض از مبدا

$105^\circ - (75^\circ + 45^\circ) = 60^\circ$ → زاویه تشکیل شده با محور x ها $= 180^\circ - 60^\circ = 120^\circ$

$\Rightarrow \tan 120^\circ = \sqrt{3} = m$

$$mb = \sqrt{3}\times\frac{a}{2} = \frac{a\sqrt{3}}{2}$$

$$f(x)=\underbrace{\cos(\pi+x)}_{=-\cos x}+\underbrace{\sin\left(\frac{\pi}{2}-x\right)}_{\cos x}\underbrace{-\tan\left(\frac{3\pi}{2}+x\right)}_{+\cot x}\overbrace{-\cot x}\Rightarrow f(x)=\cot x(9$$

$$f(x)=\cot x \rightarrow f\left(\frac{5\pi}{4}\right)=\cot(135^\circ)\rightarrow f\left(\frac{5\pi}{4}\right)=\boxed{-\sqrt{3}}$$

$$\frac{\sin\overbrace{\left(\frac{17\pi}{2}+x\right)}^{\frac{\pi}{2}+x}+\cos\overbrace{\left(\frac{15\pi}{2}-x\right)}^{\frac{3\pi}{2}-x}}{\underbrace{\cos(3\pi+x)}_{\pi+x}-\underbrace{\sin(4\pi-x)}_{2\pi-x}}=\frac{\sin\left(\frac{\pi}{2}+x\right)+\cos\left(\frac{3\pi}{2}-x\right)}{\cos(\pi+x)-\sin(2\pi-x)}=(10$$

$$\frac{\cos x-\sin x}{-\cos x+\sin x}=\frac{\cos x-\sin x}{-(\cos x-\sin x)}=\boxed{-1}$$