باران عباسی — تکلیف ۲۱ — دهم ریاضی B

۱)

$1!=1$, $2!=2$, $3!=6$, $4!=24$, $5!=120$, $6!=720$, $\dots$ ـــــ رقم یکان صفر

$1!+3!+5! = 1+6+120 \to$ رقم یکان ۷

$2!+4!+6! = 2+24+720 \Longrightarrow$ رقم یکان ۶

$6\times7=42$ ← ۲ رقم یکان

---

۲)

الف) $\sqrt[4]{\frac{1}{4+\sqrt7}} \times \sqrt[4]{(1+\sqrt7)^2} \to \sqrt[4]{\frac{1+7+2\sqrt7}{4+\sqrt7}} \to \sqrt[4]{\frac{8+2\sqrt7}{4+\sqrt7}} = \sqrt[4]{2}$

ب) $\frac{\sqrt2+\sqrt5(\sqrt{10}-2)}{10-4} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2} \to \left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)\frac{\sqrt2}{2} \to \sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}$

$\to \sqrt{6+2\sqrt5} \to \sqrt{(\sqrt5+1)^2} \to \frac{\sqrt5-1-\sqrt5-1}{2} = \frac{-2}{2} = -1$

---

۳)

الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}-2(\sqrt3-1)^{-1} \to \underbrace{\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}}-\underbrace{\frac{2}{\sqrt3-1}}$

$\frac{(2\sqrt2+3\sqrt3)(5+\sqrt6)}{25-6} = \frac{19\sqrt3+19\sqrt2}{19} \to \sqrt3+\sqrt2$ ، $\frac{2(\sqrt3+1)}{3-1}=\sqrt3+1 \Longrightarrow \sqrt3+\sqrt2-\sqrt3-1 = \sqrt2-1$

ب) $\frac{3\sqrt3-1}{4+\sqrt3}+\frac{1}{2-\sqrt3} \to \frac{(3\sqrt3-1)(4-\sqrt3)}{16-3}+\frac{2+\sqrt3}{4-3} = \frac{12\sqrt3-9-4+\sqrt3}{13}+2+\sqrt3 \to \underbrace{\frac{13\sqrt3-13}{13}}_{\sqrt3-1}+2+\sqrt3$

$\sqrt3-1+2+\sqrt3 = 2\sqrt3+1$

---

۴)

الف) $\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}-2$ ، صورت $\Rightarrow A$ ، $A^2=2\sqrt3+2\sqrt2 \to A=\sqrt2\left(\sqrt{\sqrt3+\sqrt2}\right)$

$\to \frac{\sqrt2\left(\sqrt{\sqrt3+\sqrt2}\right)}{\sqrt{\sqrt3+\sqrt2}}-2 \to \sqrt2\left(\sqrt{\frac{\sqrt3+\sqrt2}{\sqrt3-\sqrt2}}\right)-2 = \sqrt2\left(\sqrt{(\sqrt3+\sqrt2)^2}\right)-2 \to \sqrt6+2-2=\sqrt6$

ب) $2^{\frac{8}{12}}\times3\times2^{\frac{1}{4}}\times\sqrt[12]{2^{13}} \to 3\times2^{\frac{8}{12}+\frac{3}{12}+\frac{13}{12}=\frac{24}{12}} \to 2^2\times3=12$

---

۵)

$\frac{3^x(1+3+3^2+\dots+3^5)}{2^{x-2}(1+2+2^2+\dots+2^5)} = \frac{3^x(\cancel{364})}{2^{x-2}\times63} = \frac{3^x\times7}{2^{x-2}} \to \frac{3^x\times7}{2^{x-2}\times63} = \frac{3^x\times3^{-2}}{2^{x-2}} \to \frac{3^{x-2}}{2^{x-2}}=1$

$\to x-2=0$

$x=2$

6

$$((a-b)2)^2((a+b)2)^2=(a^2-b^2)^4 \longrightarrow \left(\sqrt{\sqrt{6}-2}-\sqrt{\sqrt{6}+2}\right)^4=\left(\sqrt{6}-2+\sqrt{6}+2-2\sqrt{(\sqrt{6}-2)(\sqrt{6}+2)}\right)^2$$

$$(2\sqrt{6}-2\sqrt{2})^2=\underbrace{16}_{\checkmark}(2-\sqrt{3})$$

7

$$\left(\left(a+\frac{1}{a}\right)^2-2\right)^2 \longrightarrow \left(a^2+\frac{1}{a^2}\right)^2+2-2 \longrightarrow a^4+\frac{1}{a^4}+2=2^x$$

$$a=\sqrt{7-2\sqrt{12}} \longrightarrow \sqrt{(2-\sqrt{3})^2}=\sqrt{2-\sqrt{3}}\ (2-\sqrt{3})+\frac{1}{2-\sqrt{3}}+2=2^x$$

$$\frac{1}{2-\sqrt{3}} \longrightarrow \frac{2+\sqrt{3}}{1}$$

$$7-4\sqrt{3}+7+4\sqrt{3}=14 \longrightarrow 14+2=16=2^x \longrightarrow x=4$$

8

$$A=\sqrt[5]{\sqrt[3]{2^{10}}}=2^{\frac{10}{15}}=2^{\frac{2}{3}}\times 2^{\frac{4}{3}}=2^{\frac{6}{3}}=2^2=4 \Rightarrow A$$

$$\sqrt[3]{\frac{1}{2\times 4}}=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}$$

9

$$27a^{\frac{15}{17}}=\sqrt[7]{a} \longrightarrow a \longrightarrow 27\left(3^3\times a^{\frac{15}{17}}\right)^7 \Rightarrow 3^{21}\times a^{15} \longrightarrow a^{-4}=3^{21} \longrightarrow a^{-2}=3^3$$

$$a^2=\frac{1}{27} \longrightarrow a=\frac{1}{3\sqrt{3}}$$

$$\frac{1}{3\sqrt{3}}=\frac{\sqrt{3}}{9} \qquad \frac{1}{a}=\frac{1}{\frac{\sqrt{3}}{9}}=\frac{9}{\sqrt{3}} \longrightarrow \frac{9\sqrt{3}}{3}=3\sqrt{3}$$

$$3\sqrt{3}-3 \Rightarrow 3(\sqrt{3}-1) \longrightarrow \frac{3(\sqrt{3}-1)}{1+\sqrt{3}} \longrightarrow \frac{3(\sqrt{3}-1)(\sqrt{3}-1)}{3-1} \Rightarrow \frac{3(1+3-2\sqrt{3})}{2}=\frac{\cancel{6}^{3}(2+\sqrt{3})}{\cancel{2}} \Rightarrow 6+3\sqrt{3}$$

10

$$\sqrt{x+a}=\sqrt{x-4}+2 \longrightarrow x+a=(x-4)+4\sqrt{x-4}+4 \longrightarrow x+a=x+4\sqrt{x-4} \longrightarrow \frac{a}{4}=\sqrt{x-4}$$

$$x-4=\frac{a^2}{16}$$

$$x=\frac{a^2}{16}+4$$

$$x-a=\frac{a^2}{16}+4-a \longrightarrow \sqrt{x-a}=\sqrt{\frac{a^2}{16}-a+4} \rightarrow \sqrt{\frac{a^2-16a+64}{16}}=\sqrt{\frac{(a-8)^2}{16}}=\frac{|a-8|}{4}$$

$$\sqrt{x+a}+\sqrt{x-4}-2=0 \longrightarrow \sqrt{x-4}+2+\sqrt{x-4}-2=2\sqrt{x-4}$$

$$\sqrt{x+a}=\sqrt{x-4}+2$$

$$\sqrt{x+a}+\sqrt{x-4}-2=2\sqrt{x-4} \longrightarrow 2\times\frac{a}{4}=\frac{a}{2}$$