

$\Rightarrow V \times Y_2 \subset Y_2$ ✓

(الف) $\sqrt[4]{(x+\sqrt{x})^{-1}} \times \sqrt{1+\sqrt{x}}$
 $\rightarrow \sqrt[4]{(x+\sqrt{x})^{-1}} \times \sqrt{(1+\sqrt{x})^2} \rightarrow \sqrt[4]{\frac{1}{x+\sqrt{x}}} \times \sqrt{1+2\sqrt{x}+x}$
 $= \sqrt[4]{\frac{1}{x+\sqrt{x}}} \times \sqrt{\frac{(x+\sqrt{x})^2}{x+\sqrt{x}}} = \sqrt[4]{x+\sqrt{x}}$

(ب) $\frac{\sqrt{x} + \sqrt{5}}{\sqrt{10} + 2} \times (\sqrt{x-5} - \sqrt{2+\sqrt{5}})$
 $\rightarrow \frac{\sqrt{x} + \sqrt{5}}{10 + 4 + 4\sqrt{10}} \times (3 - \sqrt{5} + 2 + \sqrt{5} - 2)$
 $= \frac{\sqrt{x} + \sqrt{5}}{14 + 4\sqrt{10}} \times (-1) = -\frac{\sqrt{x} + \sqrt{5}}{14 + 4\sqrt{10}}$

عبارت منتهی است

$-1 = \text{عبارت}$ $\rightarrow$ $+1 = \text{عبارت}$ $\rightarrow$ $1 = (\text{عبارت})^2$

1) $\frac{2\sqrt{3} + \sqrt{3}}{2 - \sqrt{3}} - 2 \times \frac{1}{\sqrt{3} - 1} \rightarrow \frac{\sqrt{3} + 1}{\sqrt{3} + 1}$
 $\frac{2\sqrt{3} + \sqrt{3}}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \rightarrow \frac{19\sqrt{3} + 19\sqrt{3}}{19}$
 $\rightarrow \frac{\sqrt{3}(\sqrt{3} + \sqrt{3})}{19} \rightarrow \frac{2\sqrt{3}}{19}$
 2) $\frac{2\sqrt{3} - 1}{2 + \sqrt{3}} + \frac{1}{2 - \sqrt{3}} \rightarrow \frac{2\sqrt{3} - 1}{2 + \sqrt{3}} + \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \rightarrow \frac{2\sqrt{3} - 1 + 2 + \sqrt{3}}{4 - 3} \rightarrow \frac{3\sqrt{3} + 1}{1} \rightarrow 3\sqrt{3} + 1$

افند $\rightarrow$
$$\frac{(\sqrt{1+\sqrt{2}} + \sqrt{\sqrt{2}-1}) (\sqrt{1+\sqrt{2}} + \sqrt{\sqrt{2}-1})}{\sqrt{\sqrt{2}-\sqrt{2}} (\cancel{\sqrt{1+\sqrt{2}} + \sqrt{\sqrt{2}-1}})} = \frac{\sqrt{2(\sqrt{2}+\sqrt{2})}}{\sqrt{(\sqrt{2}-\sqrt{2})}} \xrightarrow{\times} \frac{\sqrt{2+\sqrt{2}}}{\sqrt{\sqrt{2}+\sqrt{2}}} = \sqrt{\frac{1+\sqrt{2}}{1}}$$

$$\rightarrow \sqrt{(\sqrt{2}+2)^2} \rightarrow | \sqrt{2}+2 | - 2 \rightarrow \boxed{\sqrt{2}}$$

2) $\sqrt[12]{2^4} \times \sqrt[3]{2^4} \times \sqrt[12]{2^{13}} \rightarrow 2^{\frac{4}{12}} \times 2^{\frac{4}{3}} \times 2^{\frac{13}{12}} = 2^{\frac{4}{12} + \frac{16}{12} + \frac{13}{12}} = 2^{\frac{33}{12}} = 2^{\frac{11}{4}} = \boxed{2^{\frac{11}{4}}}$

$$\frac{r^n (1+r+q+r+1+r+r)}{r^{n-1} (1+r+r+1+r+r)} \rightarrow \frac{r^n \times r}{r^{n-1} \times q} = r \rightarrow \frac{r^n}{r^{n-1}} \times \frac{r}{q} = 1 \rightarrow \frac{r^n}{r^{n-1}} = \frac{q}{r}$$

$$\boxed{n=1} \leftarrow \left(\frac{r}{r}\right)^n = \frac{q}{r} \quad \text{D}$$

$$(\sqrt[5]{4-r} - \sqrt[5]{4+r})^5 (\sqrt[5]{4-r} + \sqrt[5]{4+r})^5 \xrightarrow{\text{bin}} ((\sqrt[5]{4-r} - \sqrt[5]{4+r})^5)^2$$

$$\rightarrow (\sqrt[5]{4-r} + \sqrt[5]{4+r} - \sqrt[5]{4-r})^5 = \frac{4^{\frac{5}{5}} + 1}{\sqrt[5]{4-r}} - 14 \sqrt[5]{\frac{4}{r}}$$

$$\rightarrow \frac{14(1-\sqrt[5]{r})}{(1-\sqrt[5]{r})} = \boxed{14} \quad \text{C}$$

$$a = \sqrt[5]{(1-\sqrt{r})^5} \rightarrow a = \sqrt[5]{1-\sqrt{r}} \rightarrow (a + \frac{1}{a} + \sqrt{r})^5 (a + \frac{1}{a} - \sqrt{r})^5 \xrightarrow{\text{bin}} =$$

$$((a + \frac{1}{a})^5 - (\sqrt{r})^5)^2 \rightarrow (1 - \sqrt{r} + \frac{1}{1-\sqrt{r}})^5 \rightarrow 1^5 \rightarrow 1^5 = 1^5$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\frac{1-\sqrt{r}}{1-\sqrt{r}} + \frac{1}{1-\sqrt{r}} + \sqrt{r} \rightarrow \frac{1-\sqrt{r}+1+\sqrt{r}}{1-\sqrt{r}} = \frac{2}{1-\sqrt{r}}$$

$$\rightarrow 1^5 = 1^5 \quad \text{C}$$

$$A = \sqrt[5]{\frac{1}{r}} \times r^{\frac{1}{5}} \rightarrow r^{\frac{1}{5}} \times r^{\frac{1}{5}} = r^{\frac{2}{5}}$$

$$(rA)^{\frac{1}{5}} \rightarrow \sqrt[5]{\frac{1}{r}} = \frac{1}{r^{\frac{1}{5}}} \quad \text{C}$$

$$\frac{1}{a^5} = r \times a^{\frac{10}{5}} \rightarrow 1 = r \times a^2 \rightarrow a^2 = \frac{1}{r} \rightarrow a = \frac{1}{\sqrt{r}}$$

$$(\frac{1}{a} - r) \rightarrow \sqrt[5]{4-r} - r \rightarrow r(\sqrt[5]{4-r} - 1) \xrightarrow{\text{bin}} \frac{1-\sqrt{r}}{1+\sqrt{r}} \rightarrow \frac{r(\sqrt[5]{4-r} - 1)(1-\sqrt{r})}{-r}$$

$$\rightarrow \frac{r(1-\sqrt{r})^5}{r} \rightarrow \frac{r(1+\sqrt{r}-2\sqrt{r})}{r} \rightarrow \frac{r(1-\sqrt{r})}{r} \rightarrow r(1-\sqrt{r}) \rightarrow 4-2\sqrt{r}$$

$$\sqrt{n+a} - \sqrt{n-r} = r \quad (\sqrt{n+a} - \sqrt{n-r})(\sqrt{n+a} + \sqrt{n-r}) = n+a - n+r = a+r$$

$$\sqrt{n+a} + \sqrt{n-r} = \frac{a+r}{r}$$

$$\downarrow$$

$$\frac{a+r}{r} - r = \frac{a}{r}$$

$$\frac{\sqrt{r} + \sqrt{a}}{\sqrt{1.} + r} = \frac{\sqrt{r} + \sqrt{a}}{\sqrt{r}(\sqrt{a} + \sqrt{r})} = \frac{1}{\sqrt{r}}$$

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$$\underbrace{(\sqrt{3-\sqrt{a}} - \sqrt{3+\sqrt{a}})}_b^2 = 3-\sqrt{a} + 3+\sqrt{a} - 2\sqrt{9-a} = 4-2 = 2$$

$$\rightarrow b^2 = 2 \xrightarrow{b < 0} b = -\sqrt{2} \rightarrow \frac{1}{\sqrt{r}} \times (-\sqrt{2}) = -1$$