

$$(11 + 31 + 51 + \dots + 251) (21 + 41 + 61 + \dots + 261) \quad (1)$$

$\begin{matrix} 11 & 31 & 51 & \dots & 251 \\ 1^2 & 3^2 & 5^2 & \dots & 25^2 \end{matrix}$
 $\begin{matrix} 21 & 41 & 61 & \dots & 261 \\ 2^2 & 4^2 & 6^2 & \dots & 26^2 \end{matrix}$

$1+2=3$ $3+2=5$ $5+2=7$ $7+2=9$ $9+2=11$ $11+2=13$ $13+2=15$ $15+2=17$ $17+2=19$ $19+2=21$ $21+2=23$ $23+2=25$

$2+24=26$

$\sqrt{x} \times \sqrt{y} = \sqrt{xy} \rightarrow \sqrt{2} = \sqrt{2}$

$$\sqrt{(4+5\sqrt{2})^{-1}} \sqrt{1+\sqrt{2}} \rightarrow \sqrt{(4+5\sqrt{2})^{-1}} \times \sqrt{(1+\sqrt{2})^2} \rightarrow \sqrt{\frac{1}{4+5\sqrt{2}} \times \frac{1+2\sqrt{2}+2}{2(4+5\sqrt{2})}} = \sqrt{2} \quad (2)$$

$$\left(\frac{\sqrt{2} + \sqrt{5}}{\sqrt{10} + 2} \right) (\sqrt{3} - \sqrt{5} - \sqrt{3} + \sqrt{5}) \xrightarrow{\text{مخرج مشترک}} \frac{2 + 5 + 2\sqrt{10}}{10 + 4 + 4\sqrt{10}} \times \underbrace{(3 - \sqrt{5} + 3 + \sqrt{5} - 3 - \sqrt{5} + 3 + \sqrt{5})}_{=1} = 1$$

$$\frac{\sqrt{8} + \sqrt{27}}{5 - \sqrt{5}} - 2(\sqrt{9} - 1)^{-1} = \frac{2\sqrt{2} + 3\sqrt{3}}{5 - \sqrt{5}} - 2 \left(\frac{1}{\sqrt{3} - 1} \right) \xrightarrow{\text{مخرج مشترک}} \frac{1}{\sqrt{3} - 1} \times \frac{\sqrt{3} + 1}{\sqrt{3} + 1} \quad (3)$$

$$\frac{2\sqrt{2} + 3\sqrt{3}}{5 - \sqrt{5}} \times \frac{5 + \sqrt{5}}{5 + \sqrt{5}} \rightarrow \frac{19\sqrt{2} + 19\sqrt{3}}{19} \rightarrow \frac{19(\sqrt{2} + \sqrt{3})}{19} \rightarrow \sqrt{2} + \sqrt{3}$$

$$\sqrt{2} + \sqrt{3} - \sqrt{3} - 1 = \sqrt{2} - 1$$

$$\frac{\sqrt{27} - 1}{4 + \sqrt{3}} + (2 - \sqrt{3})^{-1} = \frac{3\sqrt{3} - 1}{4 + \sqrt{3}} + \frac{1}{2 - \sqrt{3}} \xrightarrow{\text{مخرج مشترک}} \frac{1}{2 - \sqrt{3}} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} = \frac{2 + \sqrt{3}}{1} = 2 + \sqrt{3}$$

$$\frac{(3\sqrt{3} - 1)(2 - \sqrt{3})}{12} \times \frac{2 + \sqrt{3}}{2 + \sqrt{3}} \xrightarrow{\text{مخرج مشترک}} 12\sqrt{3} - 9 - 4 + \sqrt{3} \rightarrow 13\sqrt{3} - 13 \rightarrow 13(\sqrt{3} - 1)$$

$$\frac{\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1}}{\sqrt{\sqrt{3} - \sqrt{2}}} - 2 \rightarrow \frac{(\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1})(\sqrt{1 + \sqrt{3}} + \sqrt{\sqrt{3} - 1})}{\sqrt{\sqrt{3} - \sqrt{2}}} - 2 \quad (4)$$

$$\frac{\sqrt{2(\sqrt{3} + \sqrt{2})}}{\sqrt{\sqrt{3} - \sqrt{2}}} \times \frac{\sqrt{\sqrt{3} + \sqrt{2}}}{\sqrt{\sqrt{3} + \sqrt{2}}} = \sqrt{\frac{10 + 4\sqrt{6}}{1}} \rightarrow \sqrt{(\sqrt{2} + 2)^2} \rightarrow |\sqrt{2} + 2| - 2 = \sqrt{2}$$

$$\sqrt[3]{2^{\frac{1}{2}}} \times \sqrt[3]{16^{\frac{1}{2}}} \times \sqrt[3]{\frac{1}{2}} \rightarrow \sqrt[3]{2^{\frac{1}{2}}} \times \sqrt[3]{2^{\frac{2}{3}}} \times \sqrt[3]{2^{-\frac{1}{2}}} \rightarrow 2^{\frac{1}{6}} \times 2^{\frac{2}{6}} \times 2^{-\frac{1}{6}} = 2^{\frac{1}{6} + \frac{2}{6} - \frac{1}{6}} = 2^{\frac{2}{6}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

$$\frac{2^m (1 + 3 + 9 + 27 + 81 + 243)}{2^{m-2} (1 + 2 + 4 + 8 + 16 + 32)} = \frac{2^m \times 364}{2^{m-2} \times 63} = \frac{2^m \times 364}{2^{m-2} \times 63} = \frac{2^m \times 4 \times 91}{2^{m-2} \times 9 \times 7} = \frac{2^m \times 4 \times 13 \times 7}{2^{m-2} \times 9 \times 7} = \frac{2^m \times 4 \times 13}{2^{m-2} \times 9} = 1 \quad (5)$$

$$\frac{2^m}{2^{m-2}} = \frac{4}{1} \quad \left(\frac{2}{1} \right)^m = \frac{4}{1} \rightarrow m = 2$$

$$(\sqrt{y-2} - \sqrt{y+2})^r (\sqrt{y-2} + \sqrt{y+2})^r \rightarrow ((\sqrt{y-2} - \sqrt{y+2})^r)^r \quad (4)$$

$$\underbrace{(\sqrt{y-2} + \sqrt{y+2} - 2\sqrt{2})^r}_{2\sqrt{y-2} - 2\sqrt{y+2}} = \frac{2\sqrt{y-2} - 2\sqrt{y+2}}{2\sqrt{2}} \rightarrow \frac{14(2-\sqrt{2})}{2-\sqrt{2}} = 14 \quad (5)$$

$$a = \sqrt[5]{(2-\sqrt{2})^r} \rightarrow a = \sqrt{2-\sqrt{2}} \rightarrow (a + \frac{1}{a} + \sqrt{2})^r (a + \frac{1}{a} - \sqrt{2})^r \quad (6)$$

$$\left(\left(a + \frac{1}{a} \right)^r - (\sqrt{2})^r \right)^r \rightarrow \left(2 - \sqrt{2} + \frac{1}{2-\sqrt{2}} \right)^r \rightarrow r^r \rightarrow r^r = r^r \rightarrow r = r \quad (7)$$

$$A = \sqrt[10]{r^{10}} \times r^{\frac{r}{10}} \rightarrow r^{\frac{r}{10}} \times r^{\frac{r}{10}} \rightarrow r^r = r \quad (8)$$

$$(rA)^{-\frac{1}{r}} \rightarrow \sqrt[r]{\frac{1}{A}} = \frac{1}{r}$$

$$a^{\frac{1}{r}} = r \times a^{\frac{10}{r}} \rightarrow 1 = r \times a^{\frac{10}{r}} \rightarrow a^r = \frac{1}{r} \rightarrow a = \frac{1}{r+\sqrt{2}} \quad (9)$$

$$\left(\frac{1}{a} - r \right)^r \rightarrow r\sqrt{2} - r \rightarrow \frac{r(\sqrt{2}-1)}{1+\sqrt{2}} \times \frac{1-\sqrt{2}}{1-\sqrt{2}} = \frac{r(\sqrt{2}-1)(1-\sqrt{2})}{-1}$$

$$\frac{r(1-\sqrt{2})^r}{r} \rightarrow \frac{r(1+\sqrt{2}-2\sqrt{2})}{r} = \frac{r \times (1-\sqrt{2})}{r} = r(1-\sqrt{2}) = 2\sqrt{2} \quad (10)$$

$$\sqrt{m+a} - \sqrt{m-r} = r$$

$$\sqrt{m+a} + \sqrt{m-r} - r = ?$$

$$\frac{a+r}{r} - r = \frac{a}{r}$$

$$(\sqrt{m+a} - \sqrt{m-r}) (\sqrt{m+a} + \sqrt{m-r}) = \frac{a+r}{r}$$

$$m+a+m-r = a+r$$

