نام و نام خانوادگی: تانیا ترک ترابی　　　سری: ۲۱　　　گروه: دهم یکشنبه

۱۹٫۷۵ آفرین

۱

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!)$

$1! \to 1$ رقم یکان　　رقم یکان عبارت ۱ ← $1!+3! = 1+6 = 7$

$2! \to 2$　　رقم یکان عبارت ۲ ← $2!+4! = 2+4 = 6$

$3! \to 6$

$4! \to 4$　　$6\times7 = 42$ رقم یکان حاصل ضرب ← ۲

۲

۲

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\,\sqrt{1+\sqrt7} = \sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt{1+\sqrt7} \Rightarrow \frac{1}{4+\sqrt7}\times\frac{4-\sqrt7}{4-\sqrt7} = \frac{4-\sqrt7}{\underbrace{(4+\sqrt7)(4-\sqrt7)}_{9}}$

$\frac{\sqrt[4]{4-\sqrt7}}{\cancel{\sqrt[4]{9}}\ \sqrt3}\times\sqrt{1+\sqrt7} = \frac{\sqrt[4]{4-\sqrt7}\,\sqrt{1+\sqrt7}}{\sqrt3} = \left[\frac{(4-\sqrt7)^{\frac14}(1+\sqrt7)^{\frac24}}{3^{\frac24}}\right] = \sqrt[4]{\frac{(4-\sqrt7)(1+\sqrt7)^2}{9}} \Rightarrow \sqrt[4]{\frac{18}{9}} = \sqrt[4]{2}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\underbrace{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}_{x}\right) \Rightarrow \frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\times\frac{\sqrt{10}-2}{\sqrt{10}-2} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$

$x^2 = (\sqrt{3-\sqrt5})^2+(\sqrt{3+\sqrt5})^2-2\sqrt{(3-\sqrt5)(3+\sqrt5)} = 6-4 = 2 \Rightarrow x = \pm\sqrt2$　$\sqrt{3+\sqrt5}>\sqrt{3-\sqrt5} \Rightarrow x = -\sqrt2$　$\frac{\sqrt2}{2}\times-\sqrt2 = -\frac22 = -1$

۳

الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1} \Rightarrow \frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+15\sqrt3+\overset{4\sqrt3}{\cancel{2\sqrt{12}}}+\overset{9\sqrt2}{\cancel{3\sqrt{18}}}}{\underset{19}{25-6}} = \frac{\overset{19(\sqrt2+\sqrt3)}{19\sqrt2+19\sqrt3}}{\cancel{19}} = \sqrt2+\sqrt3$

$2(\sqrt[4]{9}-1)^{-1} = 2\times\frac{1}{\sqrt3-1}\times\frac{\sqrt3+1}{\sqrt3+1} = \cancel{2}\times\frac{\sqrt3+1}{\cancel{2}} = \sqrt3+1 \Rightarrow (\sqrt2+\cancel{\sqrt3})-\cancel{\sqrt3}-1 = \sqrt2-1$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1} \Rightarrow \frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3} = \frac{12\sqrt3\overset{-13}{\cancel{-4-9}}+\sqrt3}{\underset{13}{\cancel{16-3}}} = \frac{13\sqrt3-13}{13} = \sqrt3-1 \leftarrow \frac{\sqrt{27}-1}{4+\sqrt3}$

$(2-\sqrt3)^{-1} = \frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = \frac{2+\sqrt3}{\underset{1}{\cancel{4-3}}} = 2+\sqrt3$　$\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1} = \sqrt3-1+2+\sqrt3 = 2\sqrt3+1$

۲

۴

الف) $\frac{\overbrace{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}^{x}}{\underbrace{\sqrt{\sqrt3-\sqrt2}}_{y}} - 2 \Rightarrow x^2 = \underbrace{(\sqrt{1+\sqrt3})^2+(\sqrt{\sqrt3-1})^2}_{2\sqrt3}+\underbrace{2\sqrt{(1+\sqrt3)(\sqrt3-1)}}_{2\sqrt2} = 2\sqrt3+2\sqrt2$

$y^2 = \sqrt3-\sqrt2$　$(\text{کسر})^2 = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2} = 10+4\sqrt6 \Rightarrow \text{کسر} = \sqrt{10+4\sqrt6} = \sqrt{10+2\sqrt{24}} = \sqrt6+2$

کل عبارت: $\sqrt6+\cancel{2}-\cancel{2} = \sqrt6$

ب) $\underbrace{\sqrt[4]{\sqrt[3]{2^8}}}_{\sqrt[12]{2^8}=\sqrt[3]{4}}\times\underbrace{\sqrt[4]{162}}_{\sqrt[4]{2\times3^4}=3\sqrt[4]{2}}\times\sqrt{\sqrt[6]{4}} = 2^{\frac23}\times3^1\times2^{\frac14}\times2^1\times2^{\frac1{12}} = 2^{\frac23+\frac14+\frac1{12}}\times6 = 2^{\cancel{1}}\times6 = \cancel{12}$　$\to 2\sqrt[12]{2}$

$2\times2^{\frac1{12}}$　دقت!　$2^{\frac{8+3+1}{12}} = 2^{\frac{12}{12}} = 1$　12

۱٫۷۵

۵

$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{2^{x-2}+2^{x-1}+2^x+2^{x+1}+2^{x+2}+2^{x+3}} = 52 \Rightarrow \frac{3^x+(3^x\times3)+(3^x\times3^2)+(3^x\times3^3)+(3^x+3^4)+(3^x+3^5)}{(2^x\times2^{-2})+(2^x\times2^{-1})+2^x+(2^x\times2^1)+(2^x\times2^2)+(2^x\times2^3)} = 52$

$\Rightarrow \frac{3^x(\overbrace{1+3+3^2+3^3+3^4+3^5}^{364})}{2^x(\underbrace{2^{-2}+2^{-1}+1+2^1+2^2+2^3}_{\frac{63}{4}})} = 52 \Rightarrow \frac{3^x(364)}{2^x(\frac{63}{4})} = 52 \Rightarrow 3^x(364) = 2^x(819) \Rightarrow$

$3^x = 2^x\times(2\tfrac14) \Rightarrow \frac{3^x}{2^x} = \frac94 \Rightarrow \left(\frac32\right)^x = \frac94 \Rightarrow \boxed{x=2}$

۲

$\overbrace{(a^۲+b^۲-۲ab)^۲}^{((a-b)^۲)^۲}\overbrace{(a^۲+b^۲+۲ab)^۲}^{((a+b)^۲)^۲}=t(۲-\sqrt{۳}) \Rightarrow [(a-b)(a+b)]^۴=(a^۲-b^۲)^۴$

$a^۲=\sqrt{\sqrt{۶}-۲}$ $\quad a^۲-b^۲=\sqrt{\sqrt{۶}-۲}-\sqrt{\sqrt{۶}+۲} \Rightarrow (a^۲-b^۲)^۴=(\sqrt{\sqrt{۶}-۲}-\sqrt{\sqrt{۶}+۲})^۴=۳۲-۱۶\sqrt{۳}$

$b^۲=\sqrt{\sqrt{۶}+۲}$ ۶

$۳۲-۱۶\sqrt{۳}=t(۲-\sqrt{۳}) \Rightarrow ۱۶(\cancel{۲-\sqrt{۳}})=t(\cancel{۲-\sqrt{۳}}) \Rightarrow \boxed{t=۱۶}$ ۲

$((\sqrt{\sqrt{۶}-۲}-\sqrt{\sqrt{۶}+۲})^۲)^۲=[۲(\sqrt{۶}-\sqrt{۲})]^۲ \Rightarrow ۴(\sqrt{۶}-\sqrt{۲})^۲ \Rightarrow ۴(۸-۴\sqrt{۳})=۳۲-۱۶\sqrt{۳}$

$a=\sqrt[۴]{۷-۴\sqrt{۳}} \Rightarrow \sqrt[۴]{۳+۴-۴\sqrt{۳}} \Rightarrow \sqrt[۴]{(۲-\sqrt{۳})^۲} \Rightarrow a=\sqrt{(۲-\sqrt{۳})}$ $\quad \frac{۱}{a}=\frac{۱}{\sqrt{۲-\sqrt{۳}}} \xrightarrow{\text{گویا}} \sqrt{۲+\sqrt{۳}}$

$(\underbrace{a+\frac{۱}{a}}+\sqrt{۲})^۲(a+\frac{۱}{a}-\sqrt{۲})^۲=۲^t \Rightarrow [(a+\frac{۱}{a})^۲-(\sqrt{۲})^۲]^۲=(۶-۲)^۲=۴^۲=۱۶$ ۲

$\sqrt{۲-\sqrt{۳}}+\sqrt{۲+\sqrt{۳}}$ $\quad (a+\frac{۱}{a})^۲=(۲-\sqrt{۳})+(۲+\sqrt{۳})+۲\sqrt{(۲-\sqrt{۳})(۲+\sqrt{۳})}=۴+۲\sqrt{۱}=۶$ ۷

$۱۶=۲^t \Rightarrow \boxed{t=۴}$

$A=\sqrt[۵]{\underbrace{۴\sqrt[۳]{۲^۴}}_{\sqrt[۳]{۲^۶\times ۲^۴}}}\ \underbrace{(\frac{۱}{۲})^{-\frac{۴}{۳}}}_{(۲)^{\frac{۴}{۳}}=\sqrt[۳]{۲^۴}} \Rightarrow ۲^{\frac{۲}{۳}}\times ۲^{\frac{۴}{۳}}=۲^{\frac{۶}{۳}}=۲^۲ \Rightarrow A=۴ \Rightarrow A=۴$

$۲^{۱۰}$

$\sqrt[۱۵]{۲^{۱۰}}=\sqrt[۳]{۲^۲}$ ۸

۲

$(۲A)^{-\frac{۱}{۳}}=(۸)^{-\frac{۱}{۳}}=\frac{۱}{\sqrt[۳]{۸}}=\boxed{\frac{۱}{۲}}$

$\sqrt[۷]{a}=۲۷(a)^{\frac{۱۵}{۷}} \Rightarrow a^{\frac{۱}{۷}}=۲۷\times a^{\frac{۱۵}{۷}} \Rightarrow \frac{a^{\frac{۱}{۷}}}{a^{\frac{۱۵}{۷}}}=۲۷ \Rightarrow a^{-۲}=۲۷ \Rightarrow$

$a=\frac{۱}{۲۷^{\frac{۱}{۲}}} \Rightarrow a=\frac{۱}{\sqrt{۲۷}} \Rightarrow a=\frac{۱}{۳\sqrt{۳}}$ $\quad (\frac{۱}{a}-۳)=۳\sqrt{۳}-۳ \Rightarrow ۳(\sqrt{۳}-۱)$ ۲

$۳(\sqrt{۳}-۱)\times\frac{۱}{۱+\sqrt{۳}}=\frac{۳(\sqrt{۳}-۱)}{(۱+\sqrt{۳})}\times\frac{(۱-\sqrt{۳})}{۱-\sqrt{۳}}=\frac{۶\sqrt{۳}-۱۲}{\underset{-۲}{\cancel{۱-۳}}} \Rightarrow \frac{۲(۳\sqrt{۳}-۶)}{-۲}=-(۳\sqrt{۳}-۶)=\boxed{۶-۳\sqrt{۳}}$ ۹

$\sqrt{x+a}-\sqrt{x-۴}=۲$ $\quad \sqrt{x+a}=p,\ \sqrt{x-۴}=q \Rightarrow p-q=۲$

$\sqrt{x+a}+\sqrt{x-۴}-۲=?$ $\quad p^۲=x+a \quad q^۲=x-۴$

$p+q-۲=\frac{a+۴}{۲}-۲=\frac{a+۴-۴}{۲}=$ $\quad p^۲-q^۲=(x+a)-(x-۴)=a+۴$ ۲ ۱۰

$\boxed{\frac{a}{۲}}$ $\quad p^۲-q^۲=(p-q)(p+q) \Rightarrow (\overset{۲}{\cancel{p-q}})(p+q)=a+۴$

$۲(p+q)=a+۴ \Rightarrow p+q=\frac{a+۴}{۲}$

$\sqrt{x+a}+\sqrt{x-۴}-۲=\frac{a}{۲}$

$$\sqrt[4]{\sqrt[3]{2^{8}}} = 2^{\frac{8}{12}} = 2^{\frac{2}{3}}$$

$$\sqrt[4]{162} = \sqrt[4]{81} \times \sqrt[4]{2} = 3 \times 2^{\frac{1}{4}}$$

$$\sqrt{4\sqrt[6]{2}} = (2^{12} \times 2^{1})^{\frac{1}{12}} = 2^{\frac{13}{12}}$$

$$\left(2^{\frac{2}{3} + \frac{1}{4} + \frac{13}{12}}\right) \times 3 = 2^{\frac{8+3+13}{12}} \times 3 = 4 \times 3 = 12$$