

$$(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! + \dots + 26!)$$

$$1! \rightarrow 1 \text{ رقم یکان}$$

$$1! + 3! = 1 + 6 = 7 \rightarrow \text{رقم یکان عبارت ۷}$$

$$2! \rightarrow 2$$

$$2! + 4! = 2 + 24 = 26 \rightarrow \text{رقم یکان عبارت ۶}$$

$$3! \rightarrow 6$$

$$4! \rightarrow 24$$

$$9 \times 7 = 63 \text{ رقم یکان حاصل ضرب ۳}$$

$$\sqrt[4]{(4+\sqrt{5})^{-1}} \sqrt{1+\sqrt{5}} = \sqrt[4]{\frac{1}{4+\sqrt{5}}} \times \sqrt{1+\sqrt{5}} \Rightarrow \frac{1}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}} = \frac{4-\sqrt{5}}{(4+\sqrt{5})(4-\sqrt{5})}$$

$$\frac{\sqrt{4-\sqrt{5}}}{\sqrt{4}} \times \sqrt{1+\sqrt{5}} = \frac{\sqrt[4]{4-\sqrt{5}} \sqrt{1+\sqrt{5}}}{\sqrt{2}} = \left[\frac{(4-\sqrt{5})^{\frac{1}{4}} (1+\sqrt{5})^{\frac{1}{2}}}{2^{\frac{1}{4}}} \right] = \sqrt[4]{\frac{(4-\sqrt{5})(1+\sqrt{5})^2}{9}} = \sqrt[4]{\frac{11}{9}} = \sqrt[4]{\frac{11}{9}}$$

$$\text{ب) } \left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \right) \left(\frac{\sqrt{3}-\sqrt{5}-\sqrt{3+\sqrt{5}}}{\sqrt{10}+2} \right) \Rightarrow \frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} \times \frac{\sqrt{10}-2}{\sqrt{10}+2} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$$

$$x^2 = (\sqrt{3}-\sqrt{5})^2 + (\sqrt{3}+\sqrt{5})^2 - 2(\sqrt{3}-\sqrt{5})(\sqrt{3}+\sqrt{5}) = 9 - 4 = 5 \Rightarrow x = \pm\sqrt{5} \quad \sqrt{3+\sqrt{5}} \sqrt{3-\sqrt{5}} = 2 \Rightarrow x = \frac{-\sqrt{2}}{2} \left\{ \frac{\sqrt{2}}{2} \times -\sqrt{2} = -\frac{2}{2} = -1 \right.$$

$$\text{د) } \frac{\sqrt{8}+\sqrt{17}}{5-\sqrt{4}} - 2(\sqrt{9}-1)^{-1} \Rightarrow \frac{2\sqrt{2}+\sqrt{17}}{5-\sqrt{4}} \times \frac{5+\sqrt{4}}{5+\sqrt{4}} = \frac{10\sqrt{2}+17\sqrt{4}+2\sqrt{34}+\sqrt{68}}{25-4} = \frac{19\sqrt{2}+19\sqrt{4}}{21} = \sqrt{2}+\sqrt{4}$$

$$2(\sqrt{9}-1)^{-1} = 2 \times \frac{1}{\sqrt{9}-1} \times \frac{\sqrt{9}+1}{\sqrt{9}+1} = 2 \times \frac{\sqrt{9}+1}{8} = \frac{\sqrt{9}+1}{4} \Rightarrow (\sqrt{2}+\sqrt{4}) - \frac{\sqrt{9}+1}{4} = \sqrt{2}-1$$

$$\text{ب) } \frac{\sqrt{17}-1}{4+\sqrt{4}} + (2-\sqrt{3})^{-1} \Rightarrow \frac{2\sqrt{4}-1}{4+\sqrt{4}} \times \frac{4-\sqrt{4}}{4-\sqrt{4}} = \frac{12\sqrt{4}-4-\sqrt{4}}{16-4} = \frac{11\sqrt{4}-4}{12} = \frac{\sqrt{3}-1}{4} \leftarrow \frac{\sqrt{17}-1}{4+\sqrt{4}}$$

$$(2-\sqrt{3})^{-1} = \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = \frac{2+\sqrt{3}}{4-3} = 2+\sqrt{3} \quad \frac{\sqrt{17}-1}{4+\sqrt{4}} + (2-\sqrt{3})^{-1} = \sqrt{4}-1 + 2+\sqrt{4} = 2\sqrt{4}+1$$

$$\text{الف) } \frac{\sqrt{1+\sqrt{5}}+\sqrt{5}-1}{\sqrt{3}-\sqrt{2}} - 2 \Rightarrow x^2 = \frac{(\sqrt{1+\sqrt{5}})^2 + (\sqrt{5}-1)^2 + 2\sqrt{(1+\sqrt{5})(5-1)}}{2\sqrt{6}} = \frac{2\sqrt{5}+2\sqrt{4}}{2\sqrt{6}} = \sqrt{5}+\sqrt{4}$$

$$y = \sqrt{5}-\sqrt{2} \quad (\text{کسر})^2 = \frac{2\sqrt{5}+2\sqrt{4}}{\sqrt{5}-\sqrt{2}} \times \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = 10+4\sqrt{10} \Rightarrow \text{کسر} = \sqrt{10+4\sqrt{10}} = \sqrt{10+2\sqrt{40}} = \sqrt{4}+2$$

$$\text{ب) عبارت: } \sqrt{4} \times \sqrt{4} = \sqrt{4}$$

$$\Rightarrow \sqrt[4]{\sqrt[4]{2^8}} \times \sqrt[4]{\sqrt[4]{2^8}} \times \sqrt[4]{\sqrt[4]{2^8}} = \sqrt[4]{2^8} \times \sqrt[4]{2^8} \times \sqrt[4]{2^8} = 2^{\frac{8}{4}} \times 2^{\frac{8}{4}} \times 2^{\frac{8}{4}} = 2^2 \times 2^2 \times 2^2 = 2^6 = 64$$

$$\frac{3^x + 3^{x+1} + 3^{x+2} + 3^{x+3} + 3^{x+4} + 3^{x+5}}{2^{x-2} + 2^{x-1} + 2^x + 2^{x+1} + 2^{x+2} + 2^{x+3}} = 52 \Rightarrow \frac{3^x(1+3+3^2+3^3+3^4+3^5)}{2^x(2^{-2}+2^{-1}+1+2+2^2+2^3)} = 52$$

$$\Rightarrow \frac{3^x(1247)}{2^x(\frac{49}{2})} = 52 \Rightarrow \frac{3^x(1247)}{2^x(\frac{49}{2})} = 52 \Rightarrow 3^x(1247) = 2^x(1219) \Rightarrow$$

$$3^x = 2^x \times \left(2 \frac{1}{2}\right) \Rightarrow \frac{3^x}{2^x} = \frac{9}{4} \Rightarrow \left(\frac{3}{2}\right)^x = \frac{9}{4} \Rightarrow x = 2$$

$$\frac{(a-b)^r}{(a+b)^r} = \frac{(a^r+b^r-2ab)^r}{(a^r+b^r+2ab)^r} = \frac{1}{(1-\sqrt{e})} \Rightarrow [(a-b)(a+b)]^r = (a^r-b^r)^r$$

$$a^r = \sqrt[4]{4-r}$$

$$a^r - b^r = \sqrt[4]{4-r} - \sqrt[4]{4+r} \Rightarrow (a^r - b^r)^r = (\sqrt[4]{4-r} - \sqrt[4]{4+r})^r = 32 - 19\sqrt{r}$$

$$b^r = \sqrt[4]{4+r}$$

$$32 - 19\sqrt{r} = \frac{1}{(1-\sqrt{r})} \Rightarrow 19(1-\sqrt{r}) = 1 \Rightarrow 19 - 19\sqrt{r} = 1 \Rightarrow 18 = 19\sqrt{r} \Rightarrow \sqrt{r} = \frac{18}{19} \Rightarrow r = \left(\frac{18}{19}\right)^2$$

$$\left((\sqrt[4]{4-r} - \sqrt[4]{4+r})^r \right)^r = \left[(1-\sqrt{r})^r \right]^r \Rightarrow (1-\sqrt{r})^{r^2} = (1-\sqrt{r})^{r^2} = 32 - 19\sqrt{r}$$

$$a = \sqrt[4]{4-r} \Rightarrow \sqrt[4]{4-r} = \sqrt[4]{4-r} \Rightarrow \sqrt[4]{(1-\sqrt{r})^r} \Rightarrow a = \sqrt[4]{(1-\sqrt{r})^r}$$

$$\frac{1}{a} = \frac{1}{\sqrt[4]{(1-\sqrt{r})^r}} = \sqrt[4]{(1+\sqrt{r})^r}$$

$$\left(a + \frac{1}{a} + \sqrt{r} \right)^r \left(a + \frac{1}{a} - \sqrt{r} \right)^r = r^r \Rightarrow \left[\left(a + \frac{1}{a} \right)^r - (\sqrt{r})^r \right]^r = (4-r)^r = r^r = 19$$

$$\left(a + \frac{1}{a} \right)^r = (1-\sqrt{r})^r + (1+\sqrt{r})^r + r\sqrt{(1-\sqrt{r})(1+\sqrt{r})} = 1 + r\sqrt{1-r} = 4$$

$$19 = r^r \Rightarrow r = 19$$

$$A = \sqrt[4]{\frac{r^r}{r^r}} \left(\frac{1}{r} \right)^{-\frac{r}{r}} \Rightarrow \sqrt[4]{\frac{r^r}{r^r}} \times r^{\frac{r}{r}} = \sqrt[4]{\frac{r^r}{r^r}} \times r = r \Rightarrow A = r \Rightarrow A = r$$

$$(rA)^{-\frac{1}{r}} = (r)^{-\frac{1}{r}} = \frac{1}{\sqrt[r]{r}} = \frac{1}{r}$$

$$\sqrt[r]{a} = r\sqrt[r]{a}^{\frac{1}{r}} \Rightarrow a^{\frac{1}{r}} = r\sqrt[r]{a}^{\frac{1}{r}} \Rightarrow \frac{a^{\frac{1}{r}}}{a^{\frac{1}{r}}} = r \Rightarrow a^{-r} = r \Rightarrow$$

$$a = \frac{1}{r\sqrt[r]{r}} \Rightarrow a = \frac{1}{\sqrt[r]{r^r}} \Rightarrow a = \frac{1}{r\sqrt{r}} \quad \left(\frac{1}{a} - r \right) = \sqrt[r]{r^r} - r \Rightarrow r(\sqrt[r]{r^r} - 1)$$

$$\frac{r(\sqrt[r]{r^r} - 1)}{1+\sqrt{r}} = \frac{r(\sqrt[r]{r^r} - 1)}{(1+\sqrt{r})} \times \frac{(1-\sqrt{r})}{(1-\sqrt{r})} = \frac{r(\sqrt[r]{r^r} - 1)(1-\sqrt{r})}{1-r} \Rightarrow \frac{r(\sqrt[r]{r^r} - 1)(1-\sqrt{r})}{-r} = -(\sqrt[r]{r^r} - 1) = 1 - \sqrt[r]{r^r}$$

$$\sqrt{x+a} - \sqrt{x-r} = r$$

$$\sqrt{x+a} = p, \sqrt{x-r} = q \Rightarrow p-q = r$$

$$\sqrt{x+a} + \sqrt{x-r} - r = ?$$

$$p^r = x+a, q^r = x-r$$

$$p+q-r = \frac{a+r}{r} - r = \frac{a+r-r^2}{r} =$$

$$p^r - q^r = (x+a) - (x-r) = a+r$$

$$p^r - q^r = (p-q)(p+q) \Rightarrow (p-q)^r (p+q) = a+r$$

$$r(p+q) = a+r \Rightarrow p+q = \frac{a+r}{r}$$

$$\boxed{\frac{a}{r}}$$

$$\sqrt{x+a} + \sqrt{x-r} - r = \frac{a}{r}$$