

19, 75 آفرین

Date:

«بلیا ابیری - دهم B»

سوال 1: $(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! + \dots + 24!) =$

از آنکه به بعد به خاطر وجود عامل 2 و 2 که یکبار
هم می شود. $(1 + 3 \times 2)(2 + 4 \times 3 \times 2) = 7 \times 24 \Rightarrow$ یکبار: 26

سوال 2: $\sqrt[4]{(4+\sqrt{7})^{-1}} \sqrt{1+\sqrt{7}} = \sqrt[4]{\frac{4-\sqrt{7}}{9} \times 2(4+\sqrt{7})} = \sqrt[4]{\frac{(4-\sqrt{7})2}{9}} = \sqrt[4]{2}$
 $\hookrightarrow \sqrt[4]{\frac{1}{4+\sqrt{7}} \times \frac{4-\sqrt{7}}{4-\sqrt{7}}} = \sqrt[4]{\frac{4-\sqrt{7}}{9}}$
 $\sqrt[4]{(1+\sqrt{7})^2} = \sqrt[4]{8+2\sqrt{7}} = \sqrt[4]{2(4+\sqrt{7})}$

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right) \left(\frac{\sqrt{3}-\sqrt{5}}{2} - \sqrt{2+\sqrt{5}}\right) = \frac{\sqrt{2}}{2} \times \sqrt{2} = \frac{2}{2} = 1$

$\hookrightarrow \frac{\sqrt{2}+\sqrt{5}(\sqrt{10}-2)}{10-4} = \frac{2\sqrt{5}-2\sqrt{2}+5\sqrt{2}-2\sqrt{5}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

* $(\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}})^2 = |3-\sqrt{5}| + |3+\sqrt{5}| - 2\sqrt{9-5} = 3-\sqrt{5} + 3+\sqrt{5} - 4 = 2$
 $\Rightarrow \sqrt{2}$

سوال 3: $\frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{2}} - 2\left(\sqrt[4]{9} - 1\right)^{-1} = \sqrt{2}+\sqrt{3} - \sqrt{3}-1 = \sqrt{2}-1$

$= \frac{\sqrt{8}+\sqrt{27}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{2\sqrt{2}+3\sqrt{3}(5+\sqrt{2})}{19} = \frac{15\sqrt{2}+4\sqrt{3}+15\sqrt{3}+9\sqrt{2}}{19} = \frac{19\sqrt{2}+19\sqrt{3}}{19} = \sqrt{2}+\sqrt{3}$

* $2\left(\frac{1}{\sqrt{3}-1}\right) = 2\left(\frac{\sqrt{3}+1}{2}\right) = \sqrt{3}+1$

$$\text{ج) } \frac{3\sqrt{3}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} = \sqrt{3}-1 + 2+\sqrt{3} = 1+2\sqrt{3}$$

$$\hookrightarrow \frac{3\sqrt{3}-1(4-\sqrt{3})}{16-3} = \frac{12\sqrt{3}-9-4+\sqrt{3}}{13} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$$

$$* \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$$

$$\text{الف) } \left(\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \right)^2 - 2 = 2+\sqrt{2}-2 = \sqrt{2}$$

$$\hookrightarrow \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 4+2\sqrt{6}+2\sqrt{6}+4 \Rightarrow \sqrt{10+4\sqrt{6}} = \sqrt{(2+\sqrt{6})^2} = |2+\sqrt{6}| = 2+\sqrt{6}$$

$$\text{ج) } \sqrt[4]{3\sqrt{2}} \times \sqrt[4]{162} \times \sqrt[4]{4\sqrt{2}} = 2^{\frac{1}{4}} \times 3 \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} = 3 \times 2 = 12$$

$$S = \frac{\alpha(q^n-1)}{q-1} \rightarrow \text{ناتج: } \frac{r^n(r^y-1)}{r} \quad \text{مخرج: } \frac{r^n(r^y-1)}{1}$$

$$\Rightarrow \left(\frac{\frac{r^n(r^y-1)}{r}}{\frac{r^n(r^y-1)}{r}} \right) = \frac{r(r^n(r^y-1))}{r(r^n(r^y-1))} = \frac{r^n \times r^y}{r^{n+y}} = 1 \rightarrow n+y=0 \rightarrow \boxed{n=2}$$

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سؤال ٧:

$$\begin{aligned} a &= \sqrt[3]{\sqrt{4}-2} & b &= \sqrt[3]{\sqrt{4}+2} & (a^3+b^3-3ab)(a^3+b^3+3ab) &= \\ (a-b)^3(a+b)^3 &= (a^3-b^3)^3 &= (\sqrt{4}(\sqrt{4}-2) - \sqrt{4}(\sqrt{4}+2))^3 &= 4(2-\sqrt{4}) \\ &= (\sqrt{4}(\sqrt{4}-2 - \sqrt{4}-2))^3 &= 4(\underbrace{\sqrt{4}-\sqrt{4}-2-2}_{-4})^3 &= 4(-4)^3 = -256 \\ &= 14(2-\sqrt{4}) = 4(2-\sqrt{4}) \\ \boxed{t = 16} \end{aligned}$$

$$\begin{aligned} a &= \sqrt[3]{2-4\sqrt{3}} & (a + \frac{1}{a} + \sqrt{2})^3 (a + \frac{1}{a} - \sqrt{2})^3 &= 2^t \\ \rightarrow [(a + \frac{1}{a} + \sqrt{2})(a + \frac{1}{a} - \sqrt{2})]^3 &= [(a + \frac{1}{a})^2 - 2]^3 &= * \\ a &= \sqrt[3]{2-4\sqrt{3}} = \sqrt[3]{(2-\sqrt{3})^3} = 2-\sqrt{3} & * [(a^3 + \frac{1}{a^3})]^3 &= (\frac{-4\sqrt{3}+1}{2-\sqrt{3}})^3 = (\frac{4(2-\sqrt{3})}{2-\sqrt{3}})^3 = 64 \end{aligned}$$

$$2^t = 64 \rightarrow \boxed{t = 6}$$

$$(2A)^{-\frac{1}{3}} = t : \text{سؤال ٨}$$

$$A = \sqrt[3]{4\sqrt{14}} \left(\frac{1}{2}\right)^{-\frac{2}{3}} \rightarrow 2^{\frac{2}{3}} \times 2^{\frac{2}{3}} \times 2^{\frac{2}{3}} = 2^2 = 4$$

$$t = (2 \times 4)^{-\frac{1}{3}} = 8^{-\frac{1}{3}} = \frac{1}{2}$$

سؤال ٩:

$$\sqrt[3]{a} = (2\sqrt{a})^{\frac{10}{3}} \rightarrow a^{\frac{1}{3}} = (2\sqrt{a})^{\frac{10}{3}} \xrightarrow{\div a^{\frac{10}{3}}} a^{-2} = 2\sqrt{a} \rightarrow \sqrt{\frac{1}{a^2}} = 2\sqrt{a} \rightarrow \frac{1}{a} = 4\sqrt{a}$$

$$\frac{\frac{1}{a} - 4}{1 + \sqrt{a}} = \frac{4\sqrt{a} - 4}{1 + \sqrt{a}} \times \frac{1 - \sqrt{a}}{1 - \sqrt{a}} = \frac{4\sqrt{a} - 4 - 4 + 4\sqrt{a}}{-2} = \frac{8\sqrt{a} - 8}{-2} = -4 + 4\sqrt{a}$$

سؤال ١٥:

$$\sqrt{x+a} - \sqrt{x-f} = r \rightarrow \sqrt{x+a} - \sqrt{x-f} (\sqrt{x+a} + \sqrt{x-f}) = r (\sqrt{x+a} + \sqrt{x-f})$$

$$|x+a| - |x-f| = x+a - x+f = a+f \Rightarrow a+f = r (\sqrt{x+a} + \sqrt{x-f})$$

$$\Rightarrow \sqrt{x+a} + \sqrt{x-f} = \frac{a+f}{r} \leadsto \sqrt{x+a} + \sqrt{x-f} - r = \frac{a+f}{r} - r = \frac{a}{r}$$

$$\frac{\sqrt{r} + \sqrt{a}}{\sqrt{r} + r} = \frac{\sqrt{r} + \sqrt{a}}{\sqrt{r}(\sqrt{a} + \sqrt{r})} = \frac{1}{\sqrt{r}}$$

ب-٢

$$\underbrace{(\sqrt{3-\sqrt{a}} - \sqrt{3+\sqrt{a}})}_b^2 = 3-\sqrt{a} + 3+\sqrt{a} - 2\sqrt{9-a} = 4-2 = 2$$

$$\rightarrow b^2 = 2 \xrightarrow{b < 0} b = -\sqrt{2} \rightarrow \frac{1}{\sqrt{r}} \times (-\sqrt{2}) = -1$$