

سؤال ۱: $(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! + \dots + 24!) =$

از آنکه به بعد به خاطر وجود عامل ۲ اوله یکبار
 $(1 + 3 \times 2)(2 + 4 \times 3 \times 2) = 7 \times 24 \Rightarrow$ یکبار: ۲۶
 هم می شود.

سؤال ۲: الف) $\sqrt[4]{(4+\sqrt{5})^{-1}} \sqrt{1+\sqrt{5}} = \sqrt[4]{\frac{4-\sqrt{5}}{9} \times 2(4+\sqrt{5})} = \sqrt[4]{\frac{(4-\sqrt{5})2}{9}} = \sqrt[4]{2}$
 $\hookrightarrow \sqrt[4]{\frac{1}{4+\sqrt{5}} \times \frac{4-\sqrt{5}}{4-\sqrt{5}}} = \sqrt[4]{\frac{4-\sqrt{5}}{9}}$
 $\sqrt[4]{(1+\sqrt{5})^2} = \sqrt[4]{8+2\sqrt{5}} = \sqrt[4]{2(4+\sqrt{5})}$

ب) $\left(\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2}\right) \left(\frac{\sqrt{3}-\sqrt{5}}{4} - \sqrt{2+\sqrt{5}}\right) = \frac{\sqrt{2}}{2} \times \sqrt{2} = \frac{2}{2} = 1$

$\hookrightarrow \frac{\sqrt{2}+\sqrt{5}(\sqrt{10}-2)}{10-4} = \frac{2\sqrt{5}-2\sqrt{2}+5\sqrt{2}-2\sqrt{5}}{6} = \frac{3\sqrt{2}}{6} = \frac{\sqrt{2}}{2}$

* $(\sqrt{3}-\sqrt{5} - \sqrt{3+\sqrt{5}})^2 = |3-\sqrt{5}| + |3+\sqrt{5}| - 2\sqrt{9-5} = 3-\sqrt{5} + 3+\sqrt{5} - 4 = 2$
 $\Rightarrow \sqrt{2}$

سؤال ۳: الف) $\frac{\sqrt{8}+\sqrt{2}\sqrt{5}}{5-\sqrt{2}} - 2\left(\sqrt[4]{9} - 1\right)^{-1} = \sqrt{2}+\sqrt{5}-\sqrt{3}-1 = \sqrt{2}-1$

$= \frac{\sqrt{8}+\sqrt{2}\sqrt{5}}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{2\sqrt{2}+3\sqrt{5}(5+\sqrt{2})}{19} = \frac{10\sqrt{2}+4\sqrt{3}+15\sqrt{5}+9\sqrt{2}}{19} = \frac{19\sqrt{2}+19\sqrt{5}}{19}$
 $= \frac{19(\sqrt{2}+\sqrt{5})}{19} = \sqrt{2}+\sqrt{5}$

* $2\left(\frac{1}{\sqrt{3}-1}\right) = 2\left(\frac{\sqrt{3}+1}{2}\right) = \sqrt{3}+1$

$$\text{ج) } \frac{3\sqrt{3}-1}{4+\sqrt{3}} + (2-\sqrt{3})^{-1} = \sqrt{3}-1 + 2+\sqrt{3} = 1+2\sqrt{3}$$

$$\hookrightarrow \frac{3\sqrt{3}-1(4-\sqrt{3})}{16-3} = \frac{12\sqrt{3}-9-4+\sqrt{3}}{13} = \frac{13\sqrt{3}-13}{13} = \sqrt{3}-1$$

$$* \frac{1}{2-\sqrt{3}} \times \frac{2+\sqrt{3}}{2+\sqrt{3}} = 2+\sqrt{3}$$

$$\text{الف) } \left(\frac{\sqrt{1+\sqrt{3}} + \sqrt{\sqrt{3}-1}}{\sqrt{\sqrt{3}-\sqrt{2}}} \right)^2 - 2 = 2+\sqrt{2}-2 = \sqrt{2}$$

$$\hookrightarrow \frac{1+\sqrt{3}+\sqrt{3}-1+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} = \frac{2\sqrt{3}+2\sqrt{2}}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}} = 4+2\sqrt{6}+2\sqrt{6}+4 \Rightarrow \sqrt{10+4\sqrt{6}} = \sqrt{(2+\sqrt{6})^2} = |2+\sqrt{6}| = 2+\sqrt{6}$$

$$\text{ج) } \sqrt[4]{3\sqrt{2}} \times \sqrt[4]{162} \times \sqrt[4]{4\sqrt{2}} = 2^{\frac{1}{4}} \times 3 \times 2^{\frac{1}{4}} \times 2^{\frac{1}{4}} \times 2 = 3 \times 4 = 12$$

$$S = \frac{a(q^n-1)}{q-1} \rightarrow \text{نات: } \frac{r^n(r^y-1)}{r} \quad \text{مخرج: } \frac{r^n(r^y-1)}{1}$$

$$\Rightarrow \left(\frac{\frac{r^n(r^y-1)}{r}}{\frac{r^n(r^y-1)}{r}} \right) = \frac{r(r^n(r^y-1))}{r(r^n(r^y-1))} = \frac{r^n \times r^y}{r^{n-y} \times r^y} = r^y \rightarrow r^n - r = 0 \rightarrow \boxed{r=2}$$

Date:

سؤال ٧:

$$\begin{aligned} a &= \sqrt[3]{\sqrt{4}-2} & b &= \sqrt[3]{\sqrt{4}+2} & (a^3+b^3-3ab)(a^3+b^3+3ab) &= \\ (a-b)^3(a+b)^3 &= (a^3-b^3)^3 &= (\sqrt{4}(\sqrt{4}-2) - \sqrt{4}(\sqrt{4}+2))^3 &= 4(2-\sqrt{4}) \\ &= (\sqrt{4}(\sqrt{4}-\sqrt{4}-\sqrt{4}-\sqrt{4}))^3 &= 4(\underbrace{4-4}_{2\sqrt{4}-2})^3 &= 4(2-\sqrt{4}) \\ &= 14(2-\sqrt{4}) &= 4(2-\sqrt{4}) \end{aligned}$$

$$\boxed{t = 16}$$

$$a = \sqrt[3]{2-4\sqrt{3}} \quad \left(a + \frac{1}{a} + \sqrt{2}\right)^3 \left(a + \frac{1}{a} - \sqrt{2}\right)^3 = 2^t \quad \text{سؤال ٨:}$$

$$\rightarrow \left[\left(a + \frac{1}{a} + \sqrt{2}\right)\left(a + \frac{1}{a} - \sqrt{2}\right)\right]^3 = \left[\left(a + \frac{1}{a}\right)^2 - 2\right]^3 =$$

$$a = \sqrt[3]{2-4\sqrt{3}} = \sqrt[3]{(2-\sqrt{3})^3} = \sqrt{2-\sqrt{3}} \quad \rightarrow \left[\left(a^3 + \frac{1}{a^3}\right)\right]^3 = \left(\frac{2-4\sqrt{3}+1}{2-\sqrt{3}}\right)^3 = \left(\frac{3(2-\sqrt{3})}{2-\sqrt{3}}\right)^3$$

$$2^t = 14 \rightarrow t = 16$$

$$\boxed{(2A)^{-\frac{1}{3}}} = t : \text{سؤال ٩}$$

$$A = \sqrt[3]{4\sqrt{14}} \left(\frac{1}{2}\right)^{-\frac{2}{3}} \rightarrow 2^{\frac{2}{3}} \times 2^{\frac{2}{3}} \times 2^{\frac{2}{3}} = 2^2 = 4$$

$$t = (2 \times 4)^{-\frac{1}{3}} = 8^{-\frac{1}{3}} = \frac{1}{2}$$

سؤال ٩:

$$\sqrt[3]{a} = (2\sqrt{a})^{\frac{10}{3}} \rightarrow a^{\frac{1}{3}} = (2\sqrt{a})^{\frac{10}{3}} \xrightarrow{\div a^{\frac{10}{3}}} a^{-2} = 2\sqrt{a} \rightarrow \sqrt{\frac{1}{a^2}} = 2\sqrt{a} \rightarrow \frac{1}{a} = 4\sqrt{3}$$

$$\frac{\frac{1}{a} - 3}{1 + \sqrt{3}} = \frac{4\sqrt{3} - 3}{1 + \sqrt{3}} \times \frac{1 - \sqrt{3}}{1 - \sqrt{3}} = \frac{4\sqrt{3} - 9 - 3 + 4\sqrt{3}}{-2} = \frac{8\sqrt{3} - 12}{-2} = 4 - 4\sqrt{3}$$

سؤال ١٥:

$$\sqrt{x+a} - \sqrt{x-f} = r \rightarrow \sqrt{x+a} - \sqrt{x-f} (\sqrt{x+a} + \sqrt{x-f}) = r (\sqrt{x+a} + \sqrt{x-f})$$

$$|x+a| - |x-f| = x+a - x+f = a+f \Rightarrow a+f = r (\sqrt{x+a} + \sqrt{x-f})$$

$$\Rightarrow \sqrt{x+a} + \sqrt{x-f} = \frac{a+f}{r} \leadsto \sqrt{x+a} + \sqrt{x-f} - r = \frac{a+f}{r} - r = \frac{a}{r}$$