شقایق امامی

تکلیف شمارهٔ ۲۱ / دهم هنر 13

۱- از !۵ به بعد بخاطر وجود ۲ و ۵ یکان صفر می‌شود

$$\big(\underset{1}{1!}+\underset{6}{3!}+\underbrace{5!+\dots+25!}_{\text{یکان صفر}}\big)\big(\underset{2}{2!}+\underset{4}{4!}+\underbrace{6!+\dots+26!}_{\text{یکان صفر}}\big) \xrightarrow{\text{یکان کل عبارت}} 7\times6 = 42$$

۲- الف)

$$\sqrt[4]{(4+\sqrt7)^{-1}}\,\sqrt{1+\sqrt7} = \sqrt[4]{(4+\sqrt7)^{-1}(1+7+2\sqrt7)} = \sqrt[4]{\frac{2\cancel{(4+\sqrt7)}}{\cancel{4+\sqrt7}}} = \boxed{\sqrt[4]{2}}$$

ب)

$$\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{\cancel{\sqrt2+\sqrt5}\times\sqrt2}{\sqrt2\,\cancel{(\sqrt2+\sqrt5)}\times\sqrt2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) =$$

$$\frac{\sqrt{6-2\sqrt5}-\sqrt{6+2\sqrt5}}{2} = \frac{(\sqrt5-1)-(\sqrt5+1)}{2} = \frac{-2}{2} = \boxed{-1}$$

۳- الف)

$$\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1} = \frac{2\sqrt2+3\sqrt3}{5-\sqrt6} - \frac{2}{\sqrt3-1} \xrightarrow{\text{گویا می‌کنیم}}$$

$$\frac{(2\sqrt2+3\sqrt3)(5\sqrt6)}{19} - (\sqrt3+1) = \frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19} - (\sqrt3+1) =$$

$$\sqrt2+\cancel{\sqrt3}-\cancel{\sqrt3}-1 = \boxed{\sqrt2-1}$$

ب)

$$\frac{3\sqrt3-1}{4+\sqrt3} + (2-\sqrt3)^{-1} \xrightarrow{\text{گویا می‌کنیم}} \frac{(3\sqrt3-1)(4-\sqrt3)}{13} + 2+\sqrt3 =$$

$$\frac{12\sqrt3-4-9+\sqrt3}{13} + 2+\sqrt3 = \sqrt3-1+\sqrt3+2 = \boxed{2\sqrt3+1}$$

۴- الف)

$$\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}} - 2 = \frac{\sqrt{(1+\sqrt3)(\sqrt3+\sqrt2)}+\sqrt{\cancel{(\sqrt3-1)(\sqrt3+\sqrt2)}}}{\sqrt{3-2}}$$

$$= \sqrt{(1+\sqrt3)(\sqrt3+\sqrt2)}+\sqrt{(\sqrt3-1)(\sqrt3+\sqrt2)}$$

ب)

$$\sqrt[4]{\sqrt[3]{2^8}} \times \sqrt[4]{\frac{162}{3^4\times2}} \times \sqrt{4\sqrt[6]{2}} = 2^{\frac{8}{12}}\times3\times2^{\frac14}\times2\times2^{\frac{1}{12}} = 3\times2\times2^{\overbrace{\frac{8}{12}+\frac{3}{12}+\frac{1}{12}}^{1}}$$

$$= 3\times2\times2 = \boxed{12}$$

۵-

$$\frac{۳^x\left(\frac{۳^۶-۱}{۳-۱}\right)}{۲^{x-۲}\left(\frac{۲^۶-۱}{۲-۱}\right)}=۵۲ \longrightarrow \frac{۳^x\left(\frac{۳^۶-۱}{۲}\right)}{۲^{x-۲}\times ۶۳}=۵۲$$

$$\longrightarrow \frac{۳^x \times \cancel{۳۶۴}\times \cancel{۱۴}}{۲^{x-۲}\times \cancel{۶۳}_{۹}}=\cancel{۵۲} \longrightarrow ۳^x=۹\times ۲^{x-۲} \longrightarrow ۳^{x-۲}=۲^{x-۲} \longrightarrow \boxed{x=۲}$$

۶-

$$a=\sqrt[۴]{\sqrt{۶}-۲} \qquad b=\sqrt[۴]{\sqrt{۶}+۲}$$

$$(a^۲+b^۲-۲ab)^۲(a^۲+b^۲+۲ab)^۲=t(۲-\sqrt{۳})$$

$$(a-b)^۴(a+b)^۴=(a^۲-b^۲)^۴=\left(\sqrt{\sqrt{۲}(\sqrt{۳}-\sqrt{۲})}-\sqrt{\sqrt{۲}(\sqrt{۳}+\sqrt{۲})}\right)^۴$$

$$=\left(\sqrt[۴]{۲}\left(\sqrt{\sqrt{۳}-\sqrt{۲}}-\sqrt{\sqrt{۳}+\sqrt{۲}}\right)\right)^۴=۲\left(\sqrt{۳}-\sqrt{۲}+\sqrt{۳}+\sqrt{۲}-۲\right)=۲(۲\sqrt{۳}-۲)^۲$$

$$=۸(\underbrace{۳+۱-۲\sqrt{۳}}_{۴-۲\sqrt{۳}})=۱۶(۲-\sqrt{۳})=t(۲-\sqrt{۳}) \longrightarrow \boxed{t=۱۶}$$

۷-

$$a=\sqrt[۴]{۷-۴\sqrt{۳}} \qquad \left(a+\frac{۱}{a}+\sqrt{۲}\right)^۲\left(a+\frac{۱}{a}-\sqrt{۲}\right)^۲=۲^t$$

$$\hookrightarrow=\sqrt{۲-\sqrt{۳}} \qquad \hookrightarrow=\left(\left(a+\frac{۱}{a}\right)^۲-۲\right)^۲=\left(\underbrace{a^۲+\frac{۱}{a^۲}}_{\frac{a^۴+۱}{a^۲}}+۲-۲\right)^۲$$

$$\longrightarrow\left(\frac{۷-۴\sqrt{۳}+۱}{۲-\sqrt{۳}}\right)^۲=\left(\frac{۴\cancel{(۲-\sqrt{۳})}}{\cancel{(۲-\sqrt{۳})}}\right)^۲=۱۶=۲^t \longrightarrow \boxed{t=۴}$$

۸-

$$A=\sqrt[۵]{۴\sqrt[۳]{۱۶}}\left(\frac{۱}{۲}\right)^{-\frac{۴}{۳}}=۲^{\frac{۲}{۳}}\times ۲^{\frac{۴}{۳}}=۲^۲$$

$$(۲A)^{-\frac{۱}{۳}} \longrightarrow (۲\times ۲^۲)^{-\frac{۱}{۳}}=\boxed{\frac{۱}{۲}}$$

۹-

$$\sqrt[۷]{a}=۲۷a^{\frac{۱۵}{۷}} \longrightarrow a^{\frac{۱}{۷}}=a^{\frac{۱۵}{۷}}\times ۲۷ \longleftrightarrow a^{-۲}=۲۷ \longrightarrow a^{-۱}=۳\sqrt{۳}$$

$$\frac{\frac{۱}{a}-۳}{۱+\sqrt{۳}}=\frac{۳\sqrt{۳}-۳}{۱+\sqrt{۳}}=\frac{۳(\sqrt{۳}-۱)}{\sqrt{۳}+۱}=\frac{۳(\sqrt{۳}-۱)^۲}{۲}=\frac{۳(\overbrace{۳+۱-۲\sqrt{۳}}^{۴-۲\sqrt{۳}})}{۲}=\boxed{۶-۳\sqrt{۳}}$$

۱۰ -

$$(\sqrt{x+a}-\sqrt{x-4})(\sqrt{x+a}+\sqrt{x-4}) = \cancel{x}+a-\cancel{x}+4 = a+4$$

$$2(\sqrt{x+a}+\sqrt{x-4}) = a+4 \longrightarrow (\sqrt{x+a}+\sqrt{x-4}) = \frac{a+4}{2} = \frac{a}{2}+2$$

$$\sqrt{x+a}+\sqrt{x-4}-2 = \frac{a}{2}+2-2 = \boxed{\frac{a}{2}}$$