۱۳۵. لطفا مرتب تر بنویسید و سوالات را با دقت بیشتری حل کنید!

$A = \sqrt[5]{4^3\sqrt[3]{16}}$, $(2)^{\frac{4}{3}} \to \sqrt[3]{2^4}$ 4

(1)

$\sqrt[15]{2^4 \times 2^6}$, $\sqrt[15]{2^{30}} \Rightarrow \sqrt[15]{2^{30}} = 2^2 = \boxed{4}$

$\sqrt[7]{a} = 3\sqrt[7]{\sqrt[15]{a}} \Rightarrow \sqrt[7]{\frac{a^{1}}{a^{15}}} = 3 \to \frac{1}{a^2} = \frac{3^3}{1} \quad a = \frac{1}{3\sqrt{3}}$ 5

1.25

$\frac{3(\sqrt{3}-1)}{}$

$\frac{3\sqrt{3}-3}{1-\sqrt{3}} = \boxed{-3}$

$\sqrt{x+5} - \sqrt{x-4} = 2$ $\quad 2 + \sqrt{x-4} + \sqrt{x-4} < 2$ 40

$\to 2\sqrt{x-4}$

$\sqrt{x+5} = 2 + \sqrt{x-4}$

1.25

$2\sqrt{x-4} = \sqrt{x+5} - 4$

$\sqrt{x-4} = \frac{\sqrt{x+5}}{2} - 2$

$x - 4 = \frac{x+5}{4} + 4 - 2\sqrt{x+5}$

$x - 4 - \frac{x+5}{4} - 4 = -2\sqrt{x+5}$

$\frac{4x - 16 - x - 5 - 16}{4}$

$\frac{3x - 5 - 32}{} = -2\sqrt{x+5}$

$(3x - 5 - 32)^2 = 64x + 320$

$9x^2 - 30x + 1024 - 2(-3x \cdot 5 - 96x + 160)$

$9x^2 + 25 + 1024 + 90x + 128x - 128 = 75$

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٤. (ب)

$$\sqrt[4]{\sqrt[3]{2^{8}}} \times \sqrt[4]{16^{2}} \times \sqrt[4]{\sqrt[3]{2^{4}}} = \sqrt[12]{2^{8}} \times \sqrt[4]{2^{4}} \times \sqrt[12]{2^{4}} = 2^{\frac{8}{12}} \times 2^{\frac{4}{4}} \times 2^{\frac{4}{12}} = 2^{\frac{8}{12}+\frac{4}{4}+\frac{4}{12}} = 2^{\frac{16}{12}+\frac{4}{12}} = 2^{\frac{20}{12}} = 2^{\frac{5}{3}} = \sqrt[3]{2^{5}} = \sqrt[3]{32}$$

5. (1/5)

$$\frac{3^{x} + 3^{x+1} + 3^{x+2} + 3^{x+3} + 3^{x+4} + 3^{x+5}}{3^{x-2} + 3^{x-1} + 3^{x} + 3^{x+1} + 3^{x+2} + 3^{x+3}} = 91$$

$$\frac{3^{x}\left(1 + 3^{1} + 3^{2} + 3^{3} + 3^{4} + 3^{5}\right)}{3^{x}\left(3^{-2} + 3^{-1} + 1 + 3 + 3^{2} + 3^{3}\right)} = 91 \rightarrow \frac{364}{4}$$

$$\left(\frac{3}{2}\right)^{x} \times 2{,}9 = 91 \qquad \left(\frac{3}{2}\right)^{x} = 3{,}3$$

$$(1{,}5)^{x} \approx 3{,}3 \rightarrow x \approx 2$$

6. (1/5)

$$a = \sqrt[4]{\sqrt{5} - 2} \qquad b = \sqrt[4]{\sqrt{5} + 2}$$

$$(a+b)^{4}(a-b)^{4}$$

$$= \int \sqrt{5} - 2 + 4\left(\sqrt[4]{\sqrt{5}-2}\right)^{3}\left(\sqrt[4]{\sqrt{5}+2}\right) + 6\left(\sqrt[4]{\sqrt{5}-2}\right)^{2}\left(\sqrt[4]{\sqrt{5}+2}\right)^{2} + 4\left(\sqrt[4]{\sqrt{5}-2}\right)\left(\sqrt[4]{\sqrt{5}+2}\right)^{3} + \sqrt{5} + 2$$

$$\sqrt{5} - 2 + \left(\sqrt{5}-2\right)^{\frac{3}{4}} \times \left(\sqrt{5}+2\right)^{\frac{1}{4}} + 6\sqrt{5} + 4 + 4\left(\sqrt{5}-2\right)^{\frac{1}{4}}\left(\sqrt{5}-2\right)^{\frac{3}{4}} + \sqrt{5} + 2$$

$$2\sqrt{5} + 6\sqrt{5} + \left(\sqrt{5}-2\right)^{\frac{3}{4}} + \left(\sqrt{5}-2\right)^{\frac{1}{4}}\left(\left(\sqrt{5}+2\right)^{\frac{1}{4}} + 4\left(\sqrt{5}-2\right)^{\frac{3}{4}}\right)$$

7. (1/5)

$$a = \sqrt[4]{7 - 4\sqrt{3}} \qquad \left(\left(a + \frac{1}{a}\right)^{2} - 2\right)^{2}$$

$$\left(\frac{a^{2}+1}{a}\right)^{2} + 4 - 4\left(a + \frac{1}{a}\right)^{\frac{a^{2}+1}{a}}$$

$$\frac{a^{4} + 1 + 4a^{2} + 4a^{2}}{a^{2}} - \frac{4a^{2} + 4}{a} \rightarrow \frac{a^{4} + 1 + 4a^{2} - 4a^{2} - 4a}{a^{2}}$$

$$\frac{(a+1)^{4}}{a^{2}} \rightarrow$$

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$$\frac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۱۰}+۲} = \frac{\sqrt{۲}+\sqrt{۵}}{\sqrt{۲}(\sqrt{۵}+\sqrt{۲})} = \frac{۱}{\sqrt{۲}}$$

$$\underbrace{\left(\sqrt{۳-\sqrt{۵}} - \sqrt{۳+\sqrt{۵}}\right)}_{b}{}^{۲} = ۳-\sqrt{۵}+۳+\sqrt{۵}-۲\sqrt{۹-۵} = ۶-۴ = ۲$$

$$\rightarrow b^۲ = ۲ \xrightarrow{b<۰} b = -\sqrt{۲} \rightarrow \frac{۱}{\sqrt{۲}} \times (-\sqrt{۲}) = -۱$$

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۳ - الف

$$\frac{\sqrt{۸}+\sqrt{۲۷}}{۵-\sqrt{۶}} = \frac{۲\sqrt{۲}+۳\sqrt{۳}}{۵-\sqrt{۶}} \times \frac{۵+\sqrt{۶}}{۵+\sqrt{۶}} = \frac{۱۰\sqrt{۲}+۴\sqrt{۳}+۱۵\sqrt{۳}+۹\sqrt{۲}}{۱۹} = \sqrt{۲}+\sqrt{۳}$$

$$\left(\sqrt[۴]{۹}-۱\right)^{-۱} = \frac{۱}{\sqrt{۳}-۱} = \frac{\sqrt{۳}+۱}{۲} \rightarrow \sqrt{۲}+\sqrt{۳}-(\sqrt{۳}+۱) = \sqrt{۲}-۱$$

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۳ - ب

$$\frac{۳\sqrt{۳}-۱}{۴+\sqrt{۳}} \times \frac{۴-\sqrt{۳}}{۴-\sqrt{۳}} = \frac{۱۲\sqrt{۳}-۹-۴+\sqrt{۳}}{۱۳} = \sqrt{۳}-۱$$

$$\frac{۱}{۲-\sqrt{۳}} \times \frac{۲+\sqrt{۳}}{۲+\sqrt{۳}} = \frac{۲+\sqrt{۳}}{۱}$$

$$\sqrt{۳}-۱+۲+\sqrt{۳} = ۲\sqrt{۳}+۱$$

۴

$$\sqrt{۱+\sqrt{۳}} + \sqrt{\sqrt{۳}-۱} = A$$

$$A^۲ = ۱+\sqrt{۳}+\sqrt{۳}-۱+۲\sqrt{۳-۱} = ۲\sqrt{۳}+۲\sqrt{۲}$$

$$A = \sqrt{۲}\left(\sqrt{\sqrt{۳}+\sqrt{۲}}\right)$$

$$\frac{\sqrt{۲}\left(\sqrt{\sqrt{۳}+\sqrt{۲}}\right)}{\sqrt{\sqrt{۳}-\sqrt{۲}}} = \sqrt{۲}\sqrt{\frac{\sqrt{۳}+\sqrt{۲}}{\sqrt{۳}-\sqrt{۲}}} = \sqrt{۲}\sqrt{\frac{(\sqrt{۳}+\sqrt{۲})^۲}{۳-۲}} = \sqrt{۲}(\sqrt{۳}+\sqrt{۲}) = \sqrt{۶}+۲ \xrightarrow{-۲} \text{عبارت} = \sqrt{۶}$$

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دنباله هندسی $a=۱$ و $q=۳$

$$\frac{۳^n\left(۱+۳+۹+\ldots+۳^۵\right)}{۲^{n-۲}\left(۱+۲+۴+\ldots+۲^۵\right)} = \frac{۳^n\left(\frac{۳^۶-۱}{۲}\right)}{۲^{n-۲}\left(\frac{۲^۶-۱}{۱}\right)} = ۵۲$$

دنباله هندسی $a=۱$ و $q=۲$

$$\frac{۳^n \times ۱۳ \times ۴}{۲^{n-۲} \times ۶۳} = \left(\frac{۳}{۲}\right)^n \times \frac{۱۲ \times ۱۳}{۹} = ۵۲ \rightarrow \left(\frac{۳}{۲}\right)^n = \frac{۹}{۴} \rightarrow n = ۲$$

۶

$(a^2+b^2-2ab)^2=(a-b)^4$

$(a^2+b^2+2ab)^2=(a+b)^4 \rightarrow (a^2-b^2)^4$

$\left(\sqrt{\sqrt{4}-2}+\sqrt{\sqrt{4}+2}\right)^2=\left[\sqrt{4}-2+\sqrt{4}+2-2\sqrt{4-4}\right]^2=(2\sqrt{4}-2\sqrt{2})^2$

$24 \quad 8 \quad 8\sqrt{12}=32-14\sqrt{3}=14(2-\sqrt{3}) \rightsquigarrow \boxed{t=14}$

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$A=\sqrt[15]{4^3\times 8^2}\,(2)^{-1(-\frac{4}{2})}=(2^2)^{\frac{1}{4}}\times 2^{\frac{4}{4}}=2^{\frac{6}{4}}=4$

$2A=8 \rightsquigarrow (2A)^{-\frac{1}{4}}=\sqrt[3]{\frac{1}{8}}=\frac{1}{2}$ ب

۹

$a^{\frac{1}{7}}=27\times a^{\frac{15}{7}} \rightarrow a^{\frac{14}{7}}\times 27=1 \rightarrow a=\frac{1}{\sqrt{27}}=\frac{1}{3\sqrt{3}}=\frac{\sqrt{3}}{9}$

$\frac{1}{a}=\left(\frac{9}{\sqrt{3}}-3\right)=\frac{9-3\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}(3\sqrt{3}-3)}{\sqrt{3}}\times\frac{1}{1+\sqrt{3}}=\frac{3(\sqrt{3}-1)}{1+\sqrt{3}}\times\frac{1-\sqrt{3}}{1-\sqrt{3}}=\frac{3(4-2\sqrt{3})}{2}=6-3\sqrt{3}$

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دوعبارت داده شده در صورت سوال یادآور اتحاد مزدوج است ...!

$\left(\sqrt{x+a}-\sqrt{x-4}\right)\Big/\left(\sqrt{x+a}+\sqrt{x-4}\right)=x+a-x-(-4)=a+4$

$\quad 2 \qquad\qquad t$

$2t=a+4 \rightarrow t=\frac{a}{2}+2 \rightsquigarrow t-2=\frac{a}{2}+2-2=\frac{a}{2}$

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طبق اتحاد مزدوج $\rightsquigarrow \left((a+\frac{1}{a})^2-2\right)^2=(a^2+\frac{1}{a^2})^2=a^4+\frac{1}{a^4}+2$

$\frac{1}{7-4\sqrt{3}}\times\frac{7+4\sqrt{3}}{7+4\sqrt{3}}=\frac{7+4\sqrt{3}}{49-48}=7+4\sqrt{3}$

$a^4+\frac{1}{a^4}+2=7-4\sqrt{3}+7+4\sqrt{3}+2=16 \rightarrow 2^t=16 \rightarrow \boxed{t=4}$