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$$\left. \begin{aligned} \frac{1}{1} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{2500} &= 7 \\ \frac{1}{2} + \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{2500} &= 6 \end{aligned} \right\} \rightarrow 4+7 \rightarrow 11$$

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$$\text{الف)} \sqrt[4]{(4+\sqrt{5})^{-1}} \times \sqrt{11\sqrt{5}} = \sqrt[4]{(4+\sqrt{5})^{-1} (11\sqrt{5})} = \sqrt[4]{2(4+\sqrt{5})(4-\sqrt{5})^{-1}} \sqrt[4]{2} = \sqrt[4]{2}$$

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$$\text{ب)} \left(\frac{\sqrt{2+\sqrt{5}}}{\sqrt{1+2}} \right) (\sqrt{3-\sqrt{5}} - \sqrt{3+\sqrt{5}}) = \frac{\sqrt{2}}{\sqrt{3}} (\sqrt{2-\sqrt{5}} - \sqrt{2+\sqrt{5}}) =$$

$$\frac{\sqrt{4-2\sqrt{5}} - \sqrt{4+2\sqrt{5}}}{\sqrt{3}} = \frac{(\sqrt{5}-1) - (\sqrt{5}+1)}{\sqrt{3}} = -\frac{2}{\sqrt{3}} = \boxed{-1}$$

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$$\text{الف)} \frac{\sqrt{2}+\sqrt{3}}{2-\sqrt{2}} - \frac{2}{\sqrt{2}-1} \rightarrow \frac{(\sqrt{2}+\sqrt{3})(2+\sqrt{2})}{19} - (\sqrt{2}+1) = \frac{1-\sqrt{2}+2\sqrt{6}+2\sqrt{3}}{19} - (\sqrt{2}+1) =$$

$$= \sqrt{2}+\sqrt{3} - \sqrt{2}-1 = \boxed{\sqrt{3}-1}$$

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$$\text{ب)} \frac{\sqrt{3}-1}{4+\sqrt{2}} + \frac{1}{2-\sqrt{2}} \rightarrow \frac{(\sqrt{3}-1)(2-\sqrt{2})}{12} + 2+\sqrt{3} = \frac{12\sqrt{2}-4-9+\sqrt{6}+4\sqrt{2}}{12}$$

$$\sqrt{2}-1 + 2+\sqrt{3} = \boxed{2\sqrt{3}+1}$$

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$$\text{الف)} \frac{\sqrt{1+\sqrt{2}} + \sqrt{\sqrt{2}-1}}{\sqrt{5-\sqrt{2}}} - 2 = \frac{\sqrt{(1+\sqrt{2})(\sqrt{2}+1)} + \sqrt{(\sqrt{2}-1)(\sqrt{2}+1)}}{\sqrt{5-\sqrt{2}}} = \frac{\sqrt{(1+\sqrt{2})(\sqrt{2}+1)} + \sqrt{(\sqrt{2}-1)(\sqrt{2}+1)}}{\sqrt{5-\sqrt{2}}} =$$

$$\frac{\sqrt{4+2\sqrt{2}} + \sqrt{1-2\sqrt{2}}}{\sqrt{5-\sqrt{2}}} = \frac{\sqrt{1+2\sqrt{2}} + \sqrt{1-2\sqrt{2}}}{\sqrt{5-\sqrt{2}}} = \frac{1+\sqrt{2} - 1+\sqrt{2}}{\sqrt{5-\sqrt{2}}} = \frac{2\sqrt{2}}{\sqrt{5-\sqrt{2}}} = \sqrt{4} = \boxed{2}$$

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$$\frac{m^2 \left(\frac{m^4-1}{2-1} \right)}{2^{n-2} \left(\frac{m^4-1}{2-1} \right)} = 82 \Rightarrow \frac{m^2 \left(\frac{m^4-1}{2} \right)}{2^{n-2} \times 4m} = 82$$

$$\frac{m^2 \times 14 \times 14}{2^{n-2} \times 4 \times 4} = 82 \Rightarrow \frac{m^2}{2^{n-2}} = 9 \times 2^{n-2} \Rightarrow m^2 = 9 \times 2^{n-2} \Rightarrow n-2=0 \Rightarrow \boxed{n=2}$$

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$$(a-b)^{\frac{1}{2}}(a+b)^{\frac{1}{2}} = (a^{\frac{1}{2}}-b^{\frac{1}{2}})^{\frac{1}{2}} = \left(\sqrt{\frac{1}{2}}(\sqrt{2}-\sqrt{2}) - \sqrt{\frac{1}{2}}(\sqrt{2}+\sqrt{2}) \right)^{\frac{1}{2}} = \left(\sqrt{\frac{1}{2}}(\sqrt{2}-\sqrt{2}-\sqrt{2}-\sqrt{2}) \right)^{\frac{1}{2}}$$

$$2 \left(\frac{\sqrt{2}-\sqrt{2}+\sqrt{2}+\sqrt{2}}{2\sqrt{2}-2} \right)^{\frac{1}{2}} = 1 \left(\frac{2+1-\sqrt{2}}{2-\sqrt{2}} \right) = 14(2-\sqrt{2}) = 2(2-\sqrt{2})$$

$t = 14$ ✓

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$$a = \sqrt{v - \epsilon \sqrt{v}} = \sqrt{2 - \sqrt{2}}$$

$$\left(\left(a + \frac{1}{a} \right)^{\frac{1}{2}} - 1 \right)^{\frac{1}{2}} = \left(a^{\frac{1}{2}} + \frac{1}{a^{\frac{1}{2}}} + \frac{1}{a^{\frac{1}{2}}} \right)^{\frac{1}{2}} = \left(\frac{2 - \epsilon \sqrt{2} + 1}{2 - \sqrt{2}} \right)^{\frac{1}{2}} = \left(\frac{3 - \epsilon \sqrt{2}}{2 - \sqrt{2}} \right)^{\frac{1}{2}} = 14 = 2^t$$

$t = 14 \Rightarrow t = 2$ ✓

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$$A = \sqrt[10]{v^{\frac{1}{2}}} \quad v^{\frac{1}{2}} = v^{\frac{1}{2}} \times v^{\frac{1}{2}} = v^1$$

$$(v^{\frac{1}{2}})^{-\frac{1}{2}} = (v^1)^{-\frac{1}{2}} = \left(\frac{1}{v} \right)^{\frac{1}{2}} \quad \checkmark$$

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$$\sqrt[3]{a} = 2v \times a^{\frac{10}{3}} \Rightarrow a^{\frac{1}{3}} = a^{\frac{10}{3}} \times 2v \Rightarrow a^{-2} = 2v \Rightarrow a^{-1} = 2\sqrt{2}$$

$$\frac{\frac{1}{a} - 2}{1 + \sqrt{2}} = \frac{2\sqrt{2} - 2}{1 + \sqrt{2}} = \frac{2(\sqrt{2} - 1)}{(\sqrt{2} + 1)} = \frac{2(\sqrt{2} - 1)^2}{2} = \frac{2(2 + 1 - 2\sqrt{2})}{2} = 4 - 2\sqrt{2}$$

$4 - 2\sqrt{2}$ ✓

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$$(\sqrt{m+a} - \sqrt{m-\epsilon})(\sqrt{m+a} + \sqrt{m-\epsilon}) = m+a - m+\epsilon = a+\epsilon$$

$$2(\sqrt{m+a} + \sqrt{m-\epsilon}) = a+\epsilon \Rightarrow (\sqrt{m+a} + \sqrt{m-\epsilon}) = \frac{a+\epsilon}{2} = \frac{a}{2} + \frac{\epsilon}{2}$$

$$\sqrt{m+a} + \sqrt{m-\epsilon} - 2 = \frac{a}{2} + \frac{\epsilon}{2} - 2 = \frac{a}{2} - \frac{3}{2}$$

$\frac{a}{2}$ ✓

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