الناسیانی   تکلیف ۲۱   دهم رفتر B

①

$(1!+3!+5!+\dots+25!)(2!+4!+6!+\dots+26!) = ?$

$\frac{1 \quad 6}{7}$, صفر , $\frac{2 \quad 4}{6}$, صفر

$6\times7 \Rightarrow \boxed{2}$

②

الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\,\sqrt{1+\sqrt7}$

$\sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2} \to \sqrt[4]{\frac{1}{4+\sqrt7}}\times\sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{\frac{2(4+\sqrt7)}{4+\sqrt7}} = \boxed{\sqrt[4]{2}}$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) \to \frac{1}{\sqrt2}\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2$

$\sqrt2(\sqrt5+\sqrt2)$

$\sqrt{\frac12(3-\sqrt5+3+\sqrt5-4)} = \sqrt1 = \boxed{1}$

③

الف) $\frac{2\sqrt2+3\sqrt3}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{10\sqrt2+4\sqrt3+15\sqrt3+6\sqrt2}{19} = \frac{19(\sqrt2+\sqrt3)}{19}$

$\Rightarrow \sqrt2+\sqrt3-2\left(\frac{1}{\sqrt3-1}\right)\times\frac{\sqrt3+1}{\sqrt3+1} \to \sqrt2+\sqrt3-\sqrt3-1 = \boxed{\sqrt2-1}$

ب) $\frac{\sqrt{27}-1}{4+\sqrt3}+(2-\sqrt3)^{-1} \to \frac{3\sqrt3-1}{4+\sqrt3}\times\frac{4-\sqrt3}{4-\sqrt3} =$

$\frac{(12\sqrt3+\sqrt3-4-9)}{13} = \sqrt3-1+\left(\frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3}\right) = \boxed{1+2\sqrt3}$

④

الف) $\underbrace{\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}}_{A}-2$

$A^2 = \frac{1+\sqrt3+\sqrt3-1+2\sqrt{(1+\sqrt3)(\sqrt3-1)}}{\sqrt3-\sqrt2} = \frac{2(\sqrt3+\sqrt2)}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2}$ گویا

$= \frac{2(\sqrt3+\sqrt2)^2}{3-2}$

$A = \sqrt{2(\sqrt3+\sqrt2)^2} \to \sqrt2(\sqrt3+\sqrt2)$

$\sqrt6+2$

$\sqrt6+2-2 = \boxed{\sqrt6}$

ب) $\sqrt[4]{\sqrt[3]{28}} \times \sqrt[6]{\cancel{144}} \times \sqrt{4\sqrt[6]{2}}$

$3\sqrt[4]{2}$, 2×3^2 , $4^6\times 2 \to 2^{12}\times 2 = 2^{13}$

$\sqrt[4]{2^9} \times 3\sqrt[12]{2^3} \times \sqrt[12]{2^{13}}$

$3\times\sqrt[12]{2^9\times 2^{13}\times 2^3} = 3\times\sqrt[12]{2^{24}} = 3\times 2^2 = \boxed{12}$

(5)

$$\frac{3^x+3^{x+1}+3^{x+2}+3^{x+3}+3^{x+4}+3^{x+5}}{3^{x-2}+3^{x-1}+3^{x}+3^{x+1}+3^{x+2}+3^{x+3}} = 52 \to \frac{3^x(1+3+9+27+81+243)}{3^{x-2}(1+3+9+27+81+243)} = 52$$

364

$3^x \times 364 = 3^{x-2}\times 9\times 52$

$3^x = 2^{x-2}\times 3^2$ → $x-2=0 \to x=2$, $x=2$ ⇒ $\boxed{x=2}$

(6)

$a=\sqrt[4]{\sqrt6-2} \quad b=\sqrt[4]{\sqrt6+2}$

$(a^2+b^2-2ab)^2(a^2+b^2+2ab)^2 = t(2-\sqrt3)$

$\left[(a^2+b^2-2ab)(a^2+b^2+2ab)\right]^2 \to \left[(a-b)(a+b)\right]^4$ جایگذاری → $\boxed{16}$

$(a-b)^2 \quad (a+b)^2$, a^2-b^2

$((a-b)(a+b))^2$

$\left(\left(\sqrt{\sqrt6-2}-\sqrt{\sqrt6+2}\right)^2\right)^2$

$\sqrt6-2+\sqrt6+2-2\sqrt{6-4}$

$(2\sqrt6-2\sqrt2)^2 \to 24-8-2(2\sqrt6)(2\sqrt2)$, $8\sqrt{12}$

(7)

$a=\sqrt[4]{7-4\sqrt3} \to \sqrt[4]{(2-\sqrt3)^2} \to \sqrt{2-\sqrt3}$ ($2\times2\times\sqrt3$)

$\left(\left(a+\frac1a\right)+\sqrt2\right)^2\left(a+\frac1a-\sqrt2\right)^2 = 2^t$ ($x = a+\frac1a$)

$\left[(x+\sqrt2)(x-\sqrt2)\right]^2 \to \left[a^2+\frac1{a^2}\right]^2 \to a^4+\frac1{a^4}+2$ جایگذاری → $16=2^t$, $\boxed{t=4}$

x^2-2 , $a^2+\frac1{a^2}+2-2$

$7-4\sqrt3+\frac{1}{7-4\sqrt3}+2 \to 7-4\sqrt3+7+4\sqrt3+2 = \boxed{16}$

$\times\frac{7+4\sqrt3}{7+4\sqrt3} \to \frac{7+4\sqrt3}{1}$

(۸)

$$A = \sqrt[۵]{۴^{۳}\sqrt{۱۶}} \times \left(\frac{۱}{۴}\right)^{\frac{-۴}{۲}}$$

$$\sqrt[۱۵]{\frac{۹}{۲} \times ۲^{۴}} \qquad \frac{۲^{۴}}{۲^{۳}} \rightarrow \sqrt[۱۵]{۲^{۳۰}} = ۲^{۲} = ۴$$

$$\sqrt[۱۵]{۲^{۳۰}}$$

$$\left(\frac{۱}{۸}\right)^{-\frac{۱}{۳}} \rightarrow \left(\frac{۱}{۸}\right)^{\frac{۱}{۳}} \rightarrow \sqrt[۳]{\frac{۱}{۸}} = \boxed{\frac{۱}{۲}}$$

(۹)

$$\sqrt[۷]{a} = ۲۷ \times \sqrt[۷]{a^{۱۵}}$$

$$\frac{\sqrt[۷]{a}}{\sqrt[۷]{a^{۱۵}}} = ۲۷ \rightarrow \sqrt[۷]{\frac{a^{۱}}{a^{۱۵}}} = \sqrt[۷]{a^{-۱۴}} = ۲۷$$

$$a^{-۲} = ۲۷$$

$$\frac{۳\sqrt{۳} - ۳}{۱ + \sqrt{۳}} \times \frac{۱ - \sqrt{۳}}{۱ - \sqrt{۳}} = \boxed{۳\sqrt{۳} - ۴} \qquad \frac{۱}{a^{۲}} = ۲۷ \rightarrow \frac{۱}{a} = ۳\sqrt{۳}$$

(۱۰)

$$\sqrt{x+a} + \sqrt{x-۴} \quad - ۲ \qquad \sqrt{x+a} - \sqrt{x-۴} = ۲$$

$$\times \left(\frac{\left(\sqrt{x+a} - \sqrt{x-۴}\right)\left(\sqrt{x+a} + \sqrt{x-۴}\right)}{۲} = \frac{x + a - (x-۴)}{۲} = \frac{a+۴}{۲} - ۲ \right.$$

$$\frac{a+۴-۴}{۲} = \boxed{\frac{a}{۲}}$$