تشریح آزمون دهم B ۲ ۱۹ آفرین

$(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! \dots + 26!)$

$(1 + 3\times2)(2 + 4\times3\times2) = 7\times26 \to$ یکان $= 2$

۱) از $5!$ به بعد به خاطر وجود ۲ عامل ۲ از اینها صرفنظر می‌شود

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۱۰) الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\,\sqrt{1+\sqrt7} = \sqrt[4]{\frac{4-\sqrt7}{9}\times2(4+\sqrt7)} = \sqrt[4]{\left(\frac{16-7}{9}\right)2} = \sqrt[4]{2}$

یا $\sqrt[4]{\frac{1}{4+\sqrt7}\times\frac{4-\sqrt7}{4-\sqrt7}} = \sqrt[4]{\frac{4-\sqrt7}{9}}$

$\sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{8+2\sqrt7} = \sqrt[4]{2(4+\sqrt7)}$

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ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right) = \frac{\sqrt2}{2}\times\sqrt2 = \frac{2}{2} = 1$

$\frac{\sqrt2+\sqrt5(\sqrt{10}-2)}{10-4} = \frac{2\sqrt5-2\sqrt2+5\sqrt2-2\sqrt5}{6} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$

$* \left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2 = |3-\sqrt5| + |3+\sqrt5| - 2\sqrt{9-5} = 3-\sqrt5+3+\sqrt5-4 = 2 \to \sqrt2$

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۱۲) الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2\left(\sqrt[4]{9}-1\right)^{-1} = \sqrt2+\sqrt3-\sqrt3-1 = \sqrt2-1$

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$\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{2\sqrt2+3\sqrt3(5+\sqrt6)}{19} = \frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19} = \frac{19\sqrt2+19\sqrt3}{19} = \frac{19(\sqrt2+\sqrt3)}{19} = \sqrt2+\sqrt3$

$* \; 2\left(\frac{1}{\sqrt3-1}\right) = 2\left(\frac{\sqrt3+1}{2}\right) = \sqrt3+1$

ب) $\frac{4\sqrt3-1}{4+\sqrt3} + (2-\sqrt3)^{-1} = \sqrt3-1+2+\sqrt3 = 1+2\sqrt3$

$\frac{4\sqrt3-1(4-\sqrt3)}{16-3} = \frac{12\sqrt3-9-4+\sqrt3}{13} = \frac{13\sqrt3-13}{13} = \sqrt3-1$

$* \; \frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = 2+\sqrt3$

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۱۴) الف) $\left(\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\right)^2 - 2 = 2+\sqrt6-2 = \sqrt6$

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$\frac{|1+\sqrt3| + |\sqrt3-1| + 2\sqrt2}{\sqrt3-\sqrt2} = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2} = 6+2\sqrt6+2\sqrt6+4 \to \sqrt{10+4\sqrt6} = \sqrt{(2+\sqrt6)^2}$

$= |2+\sqrt6| = 2+\sqrt6$

ب) $\sqrt[4]{\sqrt[3]{2^9}}\times\sqrt[6]{144}\times\sqrt{4\sqrt[3]{2}} = 2^{\frac{9}{12}}\times3\times2^{\frac14}\times2^{\frac13}\times2 = 3\times4 = 12$

$S = a\left(\frac{q^n-1}{q-1}\right) \to$ صورت : $\frac{3^x(3^6-1)}{2}$ مخرج : $\frac{2^{x-2}(2^6-1)}{1}$ (۵

$\to \frac{3^x\left(\frac{3^6-1}{2}\right)}{2^{x-2}\left(\frac{2^6-1}{1}\right)} = 52 \Rightarrow \frac{3^x\left(\frac{3^6-1}{2}\right)}{2^{x-2}\times 63} = 52 \quad \frac{3^x \times \cancel{364} \times \cancel{14}}{2^{x-2} \times \cancel{567}} = \cancel{52} \Rightarrow 3^x = 9\times 2^{x-2}$ ۲

$3^{x-2} = 2^{x-2}$

$x-2=0$

$x=2$

$(a-b)^4(a+b)^4 = (a^2-b^2)^4 = \left(\sqrt{\sqrt{2}(\sqrt{3}-\sqrt{2})} - \sqrt{\sqrt{2}(\sqrt{3}+\sqrt{2})}\right)^4 = \left(\sqrt{2}\left(\sqrt{\sqrt{3}-\sqrt{2}} - \sqrt{\sqrt{3}+\sqrt{2}}\right)\right)^4$ (۶

$2\left(\frac{\cancel{\sqrt{3}}-\cancel{\sqrt{2}}+\sqrt{3}+\cancel{\sqrt{2}}-2}{2\sqrt{3}-2}\right)^2 = 8\left(\frac{\cancel{3+1-2\sqrt{3}}}{4-2\sqrt{3}}\right) = 16(2-\sqrt{3}) = t(2-\sqrt{3})$ ۲

$t=16$

$a = \sqrt[4]{7-4\sqrt{3}} \qquad \left(a+\frac{1}{a}+\sqrt{2}\right)^2\left(a+\frac{1}{a}-\sqrt{2}\right)^2 = 2^t$ (۷

$\to \left[\left(a+\frac{1}{a}+\sqrt{2}\right)\left(a+\frac{1}{a}-\sqrt{2}\right)\right]^2 = \left[\underbrace{\left(a+\frac{1}{a}\right)^2}_{a^2+\frac{1}{b^2}+\cancel{2}}-\cancel{2}\right]^2 \Rightarrow \left(a^2+\frac{1}{a^2}\right)^2 = 2-\sqrt{3}+\frac{1}{2-\sqrt{3}} = 2-\cancel{\sqrt{3}}+2+\cancel{\sqrt{3}}$ ۱,۵

$= 4$

$a = \sqrt[4]{7-4\sqrt{3}} = \sqrt[4]{(2-\sqrt{3})^2} = \sqrt{2-\sqrt{3}}$

$2^t = 4$

$t=2$

$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac{1}{2}\right)^{-\frac{4}{3}} \Rightarrow 2^{\frac{2}{5}} \times 2^{\frac{4}{15}} \times 2^{\frac{4}{3}} = 2^2 = 4 \qquad (2A)^{-\frac{1}{3}} = t$ (۸

$t = (2\times 4)^{-\frac{1}{3}} = 8^{-\frac{1}{3}} = \frac{1}{2}$ ۲

$\sqrt[7]{a} = (27a)^{\frac{15}{7}} \rightsquigarrow a^{\frac{1}{7}} = (27a)^{\frac{15}{7}} \xrightarrow{\div a^{\frac{15}{7}}} a^{-2} = 27 \to \sqrt{\frac{1}{a^2}} = \sqrt{27} \Rightarrow \frac{1}{a} = 3\sqrt{3}$ (۹

$\frac{\frac{1}{a}-3}{1-\sqrt{3}} = \frac{3\sqrt{3}-3}{1+\sqrt{3}} \times \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{3\sqrt{3}-9-3+3\sqrt{3}}{-2} = \frac{6\sqrt{3}-12}{-2} = 6-3\sqrt{3}$ ۲

$\sqrt{x+a} - \sqrt{x-4} = 2 \to \sqrt{x+a}-\sqrt{x-4}\left(\sqrt{x+a}+\sqrt{x-4}\right) = 2\left(\sqrt{x+a}+\sqrt{x+4}\right)$ (۱۰

$|x+a| - |x-4| = x+a-x+4 = a+4 \Rightarrow a+4 = 2\left(\sqrt{x+a}+\sqrt{x-4}\right)$ ۲

$\Rightarrow \underbrace{\sqrt{x+a}+\sqrt{x-4}}_{-2} = \frac{a+4}{2} - 2 \Rightarrow \frac{a}{2}$

۲ـ الف

$$\sqrt[4]{(4+\sqrt{7})^{-1}} \times \sqrt[4]{(1+\sqrt{7})^{2}} = \frac{\sqrt[4]{7+1+2\sqrt{7}}}{\sqrt[4]{4+\sqrt{7}}} = \sqrt[4]{\frac{8+2\sqrt{7}}{4+\sqrt{7}}} = \sqrt[4]{2}$$

۲ـ ب

$$\frac{\sqrt{2}+\sqrt{5}}{\sqrt{10}+2} = \frac{\sqrt{2}+\sqrt{5}}{\sqrt{2}(\sqrt{5}+\sqrt{2})} = \frac{1}{\sqrt{2}}$$

$$(\underbrace{\sqrt{3-\sqrt{5}}-\sqrt{3+\sqrt{5}}}_{b})^{2} = 3-\sqrt{5}+3+\sqrt{5}-2\sqrt{9-5} = 6-4 = 2$$

$$\longrightarrow b^{2}=2 \xrightarrow{b<0} b=-\sqrt{2} \longrightarrow \frac{1}{\sqrt{2}}\times(-\sqrt{2}) = -1$$

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طبق اتحاد مزدوج $\rightsquigarrow \left((a+\frac{1}{a})^{2}-2\right)^{2} = (a^{2}+\frac{1}{a^{2}})^{2} = a^{4}+\frac{1}{a^{4}}+2$

$$\frac{1}{7-4\sqrt{3}} \times \frac{7+4\sqrt{3}}{7+4\sqrt{3}} = \frac{7+4\sqrt{3}}{49-48} = 7+4\sqrt{3}$$

$$a^{4}+\frac{1}{a^{4}}+2 = 7-4\sqrt{3}+7+4\sqrt{3}+2 = 16 \longrightarrow 2^{t}=16 \longrightarrow \boxed{t=4}$$