[illegible] دهم B ۱۱

$$(1! + 3! + 5! + \dots + 25!)(2! + 4! + 6! + \dots + 26!)$$

۱) از $5!$ به بعد به خاطر وجود ۲ عامل ۲ و ۵ یکان صفر می‌شود

$$(1 + 3\times 2)(2 + 4\times 3\times 2) = 7\times 26 \to \text{یکان} = 2$$

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۱۰) الف) $\sqrt[4]{(4+\sqrt7)^{-1}}\ \sqrt{1+\sqrt7} = \sqrt[4]{\frac{4-\sqrt7}{9}\times 2(4+\sqrt7)} = \sqrt[4]{\frac{(16-7)2}{9}} = \sqrt[4]{2}$

یا $\sqrt[4]{\frac{1}{4+\sqrt7}\times\frac{4-\sqrt7}{4-\sqrt7}} = \sqrt[4]{\frac{4-\sqrt7}{9}}$

$$\sqrt[4]{(1+\sqrt7)^2} = \sqrt[4]{8+2\sqrt7} = \sqrt[4]{2(4+\sqrt7)}$$

ب) $\left(\frac{\sqrt2+\sqrt5}{\sqrt{10}+2}\right)\left(\underbrace{\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}}_{*}\right) = \frac{\sqrt2}{2}\times\sqrt2 = \frac{2}{2} = 1$

$$\frac{\sqrt2+\sqrt5(\sqrt{10}-2)}{10-4} = \frac{2\sqrt5-2\sqrt2+5\sqrt2-2\sqrt5}{6} = \frac{3\sqrt2}{6} = \frac{\sqrt2}{2}$$

$$* \quad \left(\sqrt{3-\sqrt5}-\sqrt{3+\sqrt5}\right)^2 = |3-\sqrt5| + |3+\sqrt5| - 2\sqrt{9-5} = 3-\sqrt5+3+\sqrt5-4 = 2 \to \sqrt2$$

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۱۳) الف) $\frac{\sqrt8+\sqrt{27}}{5-\sqrt6} - 2(\sqrt[4]{9}-1)^{-1} = \sqrt2+\sqrt3-\sqrt3-1 = \sqrt2-1$

$$\frac{\sqrt8+\sqrt{27}}{5-\sqrt6}\times\frac{5+\sqrt6}{5+\sqrt6} = \frac{2\sqrt2+3\sqrt3(5+\sqrt6)}{19} = \frac{10\sqrt2+4\sqrt3+15\sqrt3+9\sqrt2}{19} = \frac{19\sqrt2+19\sqrt3}{19} = \frac{19(\sqrt2+\sqrt3)}{19} = \sqrt2+\sqrt3$$

$$* \quad 2\left(\frac{1}{\sqrt3-1}\right) = 2\left(\frac{\sqrt3+1}{2}\right) = \sqrt3+1$$

ب) $\frac{3\sqrt3-1}{4+\sqrt3} + (2-\sqrt3)^{-1} = \sqrt3-1+2+\sqrt3 = 1+2\sqrt3$

$$\frac{3\sqrt3-1(4-\sqrt3)}{16-3} = \frac{12\sqrt3-9-4+\sqrt3}{13} = \frac{13\sqrt3-13}{13} = \sqrt3-1$$

$$* \quad \frac{1}{2-\sqrt3}\times\frac{2+\sqrt3}{2+\sqrt3} = 2+\sqrt3$$

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۱۴) الف) $\left(\frac{\sqrt{1+\sqrt3}+\sqrt{\sqrt3-1}}{\sqrt{\sqrt3-\sqrt2}}\right)^2 - 2 = 2+\sqrt6-2 = \sqrt6$

$$\frac{|1+\sqrt3|+|\sqrt3-1|+2\sqrt2}{\sqrt3-\sqrt2} = \frac{2\sqrt3+2\sqrt2}{\sqrt3-\sqrt2}\times\frac{\sqrt3+\sqrt2}{\sqrt3+\sqrt2} = 6+2\sqrt6+2\sqrt6+4 \to \sqrt{10+4\sqrt6} = \sqrt{(2+\sqrt6)^2} = |2+\sqrt6| = 2+\sqrt6$$

ب) $\sqrt[4]{\sqrt[3]{2^8}} \times \sqrt[4]{144} \times \sqrt{4\sqrt[3]{2}} = 2^{\frac{8}{12}} \times 3 \times 2^{\frac14} \times 2^{\frac{1}{12}} \times 2 = 12 \times 4 = 13$

$S = a\left(\frac{q^n-1}{q-1}\right) \to$ صورت: $\frac{3^x(3^4-1)}{2}$ مخرج: $\frac{2^{x-2}(2^4-1)}{1}$ (5

$\to \frac{3^x\left(\frac{3^4-1}{2}\right)}{2^{x-2}\left(\frac{2^4-1}{1}\right)} = 52 \Rightarrow \frac{3^x\left(\frac{3^4-1}{2}\right)}{2^{x-2}\times 15} = 52 \quad \frac{3^x \times 40}{2^{x-2}\times 15} = 52 \Rightarrow 3^x = 9\times 2^{x-2}$

$3^{x-2} = 2^{x-2}$

$x-2=0$

$x=2$

$(a-b)^4(a+b)^4 = (a^2-b^2)^4 = \left(\sqrt{\sqrt2(\sqrt3-\sqrt2)} - \sqrt{\sqrt2(\sqrt3+\sqrt2)}\right)^4 = \left(\sqrt2\left(\sqrt{\sqrt3-\sqrt2}-\sqrt{\sqrt3+\sqrt2}\right)\right)^4$ (6

$2\left(\frac{\cancel{\sqrt3}-\cancel{\sqrt2}+\sqrt3+\cancel{\sqrt2}-2}{2\sqrt3-2}\right)^2 = 8\left(\frac{\cancel{3+1-2\sqrt3}}{4-2\sqrt3}\right) = 16(2-\sqrt3) = t(2-\sqrt3)$

$t=16$

$a = \sqrt[4]{7-4\sqrt3} \quad \left(a+\frac1a+\sqrt2\right)^2\left(a+\frac1a-\sqrt2\right)^2 = 2^t$ (7

$\to \left[\left(a+\frac1a+\sqrt2\right)\left(a+\frac1a-\sqrt2\right)\right]^2 = \left[\left(a+\frac1a\right)^2-\cancel{2}\right]^2 \Rightarrow \left(a^2+\frac1{a^2}\right)^2 = 2-\sqrt3+\frac{1}{2-\sqrt3} = 2-\cancel{\sqrt3}+2+\cancel{\sqrt3}$

$a = \sqrt[4]{7-4\sqrt3} = \sqrt[4]{(2-\sqrt3)^2} = \sqrt{2-\sqrt3} \quad a^2+\frac{1}{b^2}\cancel{+2}$ $=4$

$2^t=4$

$t=2$

$A = \sqrt[5]{4\sqrt[3]{16}}\left(\frac12\right)^{-\frac23} \Rightarrow 2^{\frac25}\times 2^{\frac4{15}}\times 2^{\frac43} = 2^2=4 \quad (2A)^{-\frac13} = t$ (8

$t = (2\times4)^{-\frac13} = 8^{-\frac13} = \frac12$

$\sqrt[7]{a} = (27a)^{\frac{15}{7}} \sim a^{\frac17} = (27a)^{\frac{15}{7}} \xrightarrow{\div a^{\frac{15}{7}}} a^{-2} = 27 \to \sqrt{\frac1{a^2}} = \sqrt{27} \Rightarrow \frac1a = 3\sqrt3$ (9

$\frac{\frac1a-3}{1-\sqrt3} = \frac{3\sqrt3-3}{1+\sqrt3}\times\frac{1-\sqrt3}{1-\sqrt3} = \frac{3\sqrt3-9-3+3\sqrt3}{-2} = \frac{6\sqrt3-12}{-2} = 6-3\sqrt3$

$\sqrt{x+a}-\sqrt{x-4} = 2 \to \sqrt{x+a}-\sqrt{x-4}\left(\sqrt{x+a}+\sqrt{x-4}\right) = 2\left(\sqrt{x+a}+\sqrt{x+4}\right)$ (10

$|x+a|-|x-4| = x+a-x+4 = a+4 \Rightarrow a+4 = 2\left(\sqrt{x+a}+\sqrt{x-4}\right)$

$\Rightarrow \sqrt{x+a}+\sqrt{x-4}-2 = \frac{a+4}{2}-2 \Rightarrow \frac a2$